# Generation of Influence Lines Using the Muller-Breslau Principle

**Objectives of the materials covered:** The student should be able to use the Mueller-Breslau Principle to construct an influence line for reactions, shear, and moment at any point in a simply-supported beam.

**Introduction:** In 1886, Muller-Breslau developed a way to easily draw influence lines by applying a unit distortion to the structure. His principal stated that the height of an influence line for the reaction, or for the axial force, shear, or moment at any point in the structure is proportional to the deflected shape of that structure, when the structure is subjected to a unit distortion in the direction of that reaction, axial force, shear, or moment.

**Influence lines for reactions:** As an example, an influence line for the left reaction of a beam can be generated by placing a unit vertical displacement at the reaction point, in the direction of that reaction. Note that this gives the same result as was obtained by statics in our earlier discussion. However, note that it only took 2 seconds to get it this way.

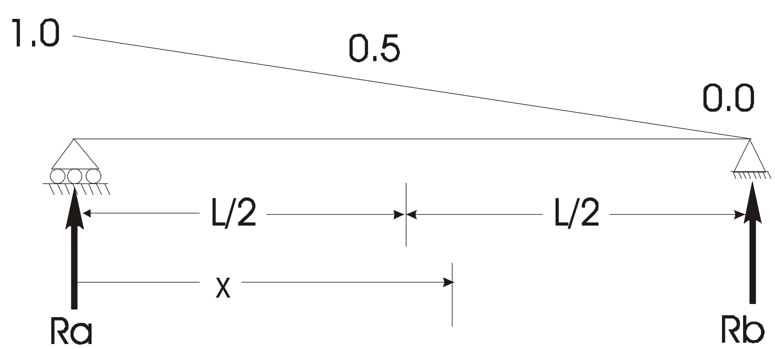



**Figure 1**

By the same method, an influence line for the reaction at B can be found by simply distorting the reaction at B upwards a unit distortion, and allowing the structure to form its own influence line.

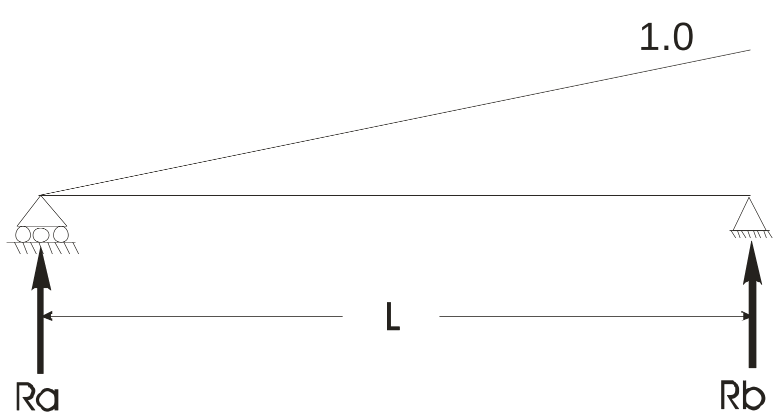



**Figure 2**

**Influence lines for shear:** To generate an influence line for shear at the quarter point in the beam, the steel beam is first removed from its supports, and distorted in shear a unit amount. Note that in order to distort the beam in shear by 1.0, we temporarily remove it from the right support, otherwise holding it in position.

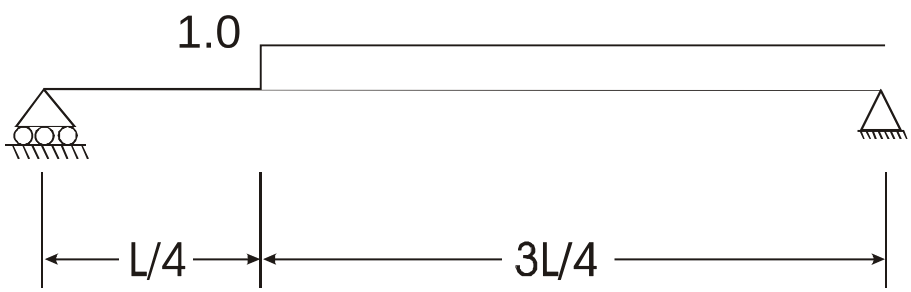



**Figure 3**

Now, realizing that the beam is supported on the left, but not on the right, we will rotate the beam clockwise until the beam again aligns with its right support, and reinsert the right-hand connecting pin.

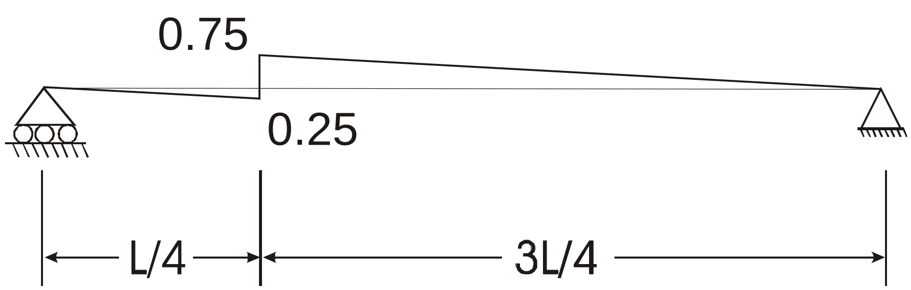



**Figure 4**

In summary, the beam was unpinned from the right reaction to permit the application of a unit shear distortion at the quarter-point. This caused a 1.0 incompatibility between the right end of the beam and its reaction. To correct this we rotated the right end of the beam down by a distance = 1.0 at which time the pin was again inserted to support the right end of the beam. Note that the beam remained pinned to the left reaction during this entire operation.

This rotation, about the left support, also caused the quarter-point of the beam to rotate down. However, since the quarter-point is not as far from the point of rotation (the left reaction) as is the right end of the beam, it will not rotate down as far. It will only rotate down y = (¼)*1.0 since it is only ¼ of the distance from the pivot point as is the right end of the beam. That is to say, it rolls down in proportion to how far it is located from the left reaction.

Thus a point immediately to the left of the quarter point, which before the rotating operation had a y value of 0.0, moves down 0.25, to the value y = -0.25. However, a point immediately to the right of the quarter point, which before the rotating operation had a y value of 1.0, also moves down 0.25, and thus will rotate to a final height of 1.0-0.25 = 0.75, as shown. All other points on the beam will move down in proportion to their distance from the pivot point, x, thus resulting in the straight-line influence line shown.

Now although this may seem complicated, it results in the influence line for the shear at the quarter with far less computation than required by the application of statics.

**Influence lines for moment:** To generate an influence line for moment at the quarter point in the beam, the steel beam is first removed from its supports, and distorted in moment by a unit amount:

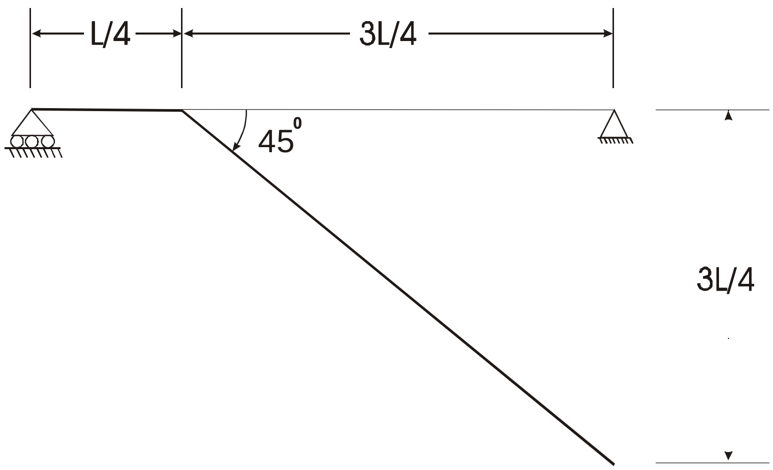



**Figure 5**

Note that in order to distort the beam in moment by 1:1 (45 degrees), we temporarily remove it from the right support, otherwise holding it in position. Now, realizing that the beam is supported on the left, but not on the right, we will rotate the beam counter-clockwise until the beam again aligns with its right support, and reinsert the right-hand connecting pin.

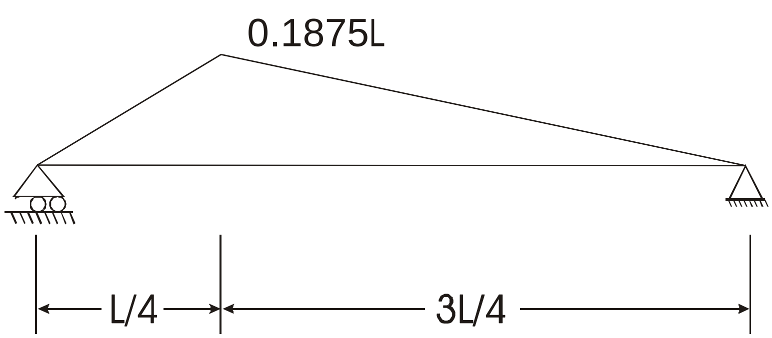



**Figure 6**

In summary, the beam was unpinned from the right reaction to permit the application of a unit moment distortion at the quarter-point. This caused a 3L/4 incompatibility between the right end of the beam and its reaction. To correct this we rotated the beam counter-clockwise about the pinned left reaction, until the right end of the beam came up by a distance = 3L/4, at which time the pin was again inserted to support the right end of the beam. Note that the beam remained pinned to the left reaction during this entire operation.

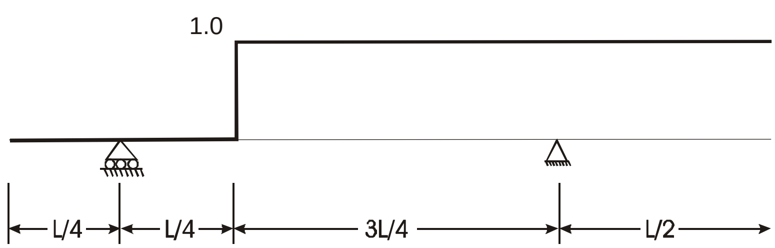
This rotation, about the left support, also caused the quarter-point of the beam to rotate up. However, since the quarter-point is not as far from the point of rotation (the left reaction) as is the right end of the beam, it will not rotate up as far. It will only rotate up y = (¼)*3L/4 since it is only ¼ of the distance from the pivot point as is the right end of the beam. That is to say, it rolls up in proportion to how far it is located from the left reaction. All other points on the beam will also move up in proportion to their distance from the pivot point, x, thus resulting in the straight-line influence line shown in Figure 6.

Again, although this may seem complicated, it results in the influence line for the moment at the quarter with far less computation than required by the application of statics.

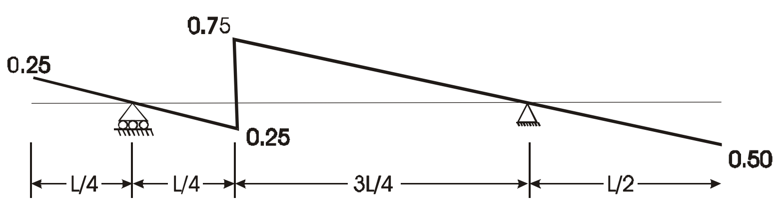
**Other Examples for influence lines:**

Influence line for shear at the quarter-point of a doubly-overhanging simply supported beam:

Initially, with a unit shear distortion applied to the beam at the quarter point:

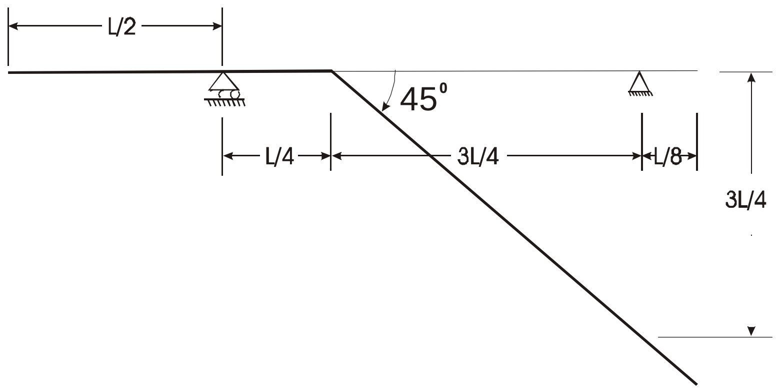



Finally, after rotating the beam and reattaching the supports:




Influence line for moment at the quarter-point of a doubly-overhanging simply supported beam:

Initially, with a unit moment distortion applied to the beam at the quarter point:




Finally, after rotating the beam and reattaching the pins in the supports: