Chapter 5

STRESS TRANSFORMATION AND MOHR'S CIRCLE

5.1 Stress Transformations and Mohr's Circle

We have now shown that, in the absence of body moments, there are six components of the stress tensor at a material point. In Cartesian coordinates, these are as shown below. For clarity, only the stress components on the positive faces are shown.

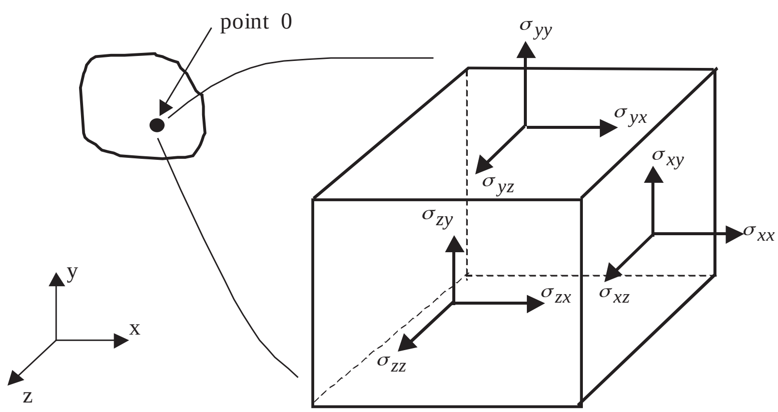


Figure 5.1: Stress Components in Cartesian Coordinates

Note that the continuum of interest may very often be irregular in shape, so that the choice of a coordinate system is quite arbitrary. Nevertheless, this choice of coordinate system will determine the planes on which the components of stress will be evaluated. These planes will not always be the planes that are physically interesting or informative, such as the planes on which cracks will grow in a solid, and there is need to be able to find the components of stress in other coordinate systems.

Consider a column loaded by a compressive load as shown below in the left figure below. Draw two different free-body diagrams. In free-body #1, a cut is made normal to the x-axis. For free-body #2, a cut is made at some angle θ to the x-axis and an x'-y' coordinate system is defined with x' at the same angle θ to the x-axis.

In free-body 1, only a normal stress σ_{yy} will exist on the cut plane (the shear stress σ_{xy} is zero). However, in free-body #2, both a normal stress $\sigma_{x'x'}$ and shear stress $\sigma_{x'y'}$ will exist on the inclined

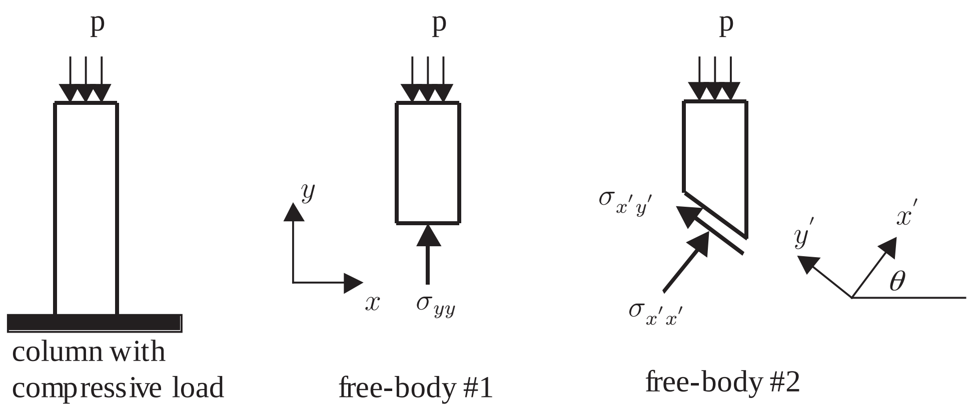


Figure 5.2: Column Loaded in Compression

plane (the traction vector on the inclined surface must have two components so that the resultant force equilibrates the applied load). Consequently, it is important to be able to define the stress on any plane with any orientation relative to the x-axis. In this case, no shear stress exists in the x-y coordinate system, but does exist in the x'-y' coordinate system. In free-body #2, $\sigma_{x'x'}$ and $\sigma_{x'y'}$ must satisfy conservation of linear momentum such that the sum of the horizontal components of the forces due to the stresses are zero, while the sum of the vertical components must equal the vertical force due to the applied traction on the column.

Thus, it is desirable to develop a method for determining the stresses on arbitrary planes at any point once σ_{xx}, σ_{yy}, σ_{zz}, σ_{yz}, σ_{xz}, and σ_{xy} have been determined. The process of finding these stresses in a coordinate system like x'-y', which is rotated by some angle θ relative to the x-axis, is called **stress transformation**.

In this text, we will consider only states of stress in which shear stresses are non-zero in at most one plane. This state of stress is termed **generalized plane stress**. An example of generalized plane stress in the x-y plane is shown below. Note that no shear stresses exist in the y-z or x-z planes for this example.

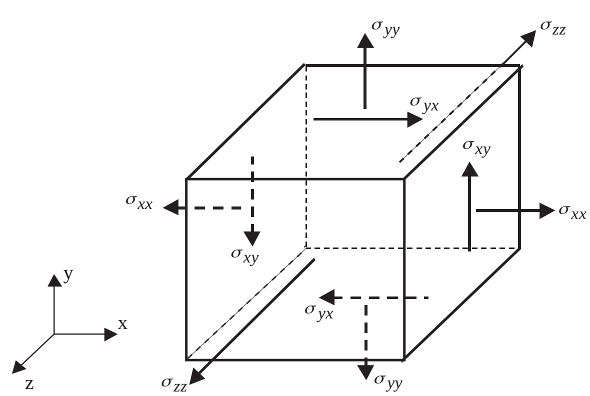


Figure 5.3: Generalized Plane Stress in x-y Plane

Principal stress is defined as the normal stress that exists on a plane (at some angle θ) where all shear stresses are equal to zero. In Figure 5.3 above, σ_{zz} is a principal stress since no shear stresses are shown on the z face (the face with unit outward normal $\mathbf{k}$). Since the coordinate system is arbitrary, note that if a state of generalized plane stress exists at a point, the coordinate system can always be rotated such that the x'-y' stress state is as shown below in Figure 5.4

Consider a solid body such as that shown below. Suppose that we start with the state of stress defined in x-y coordinates.

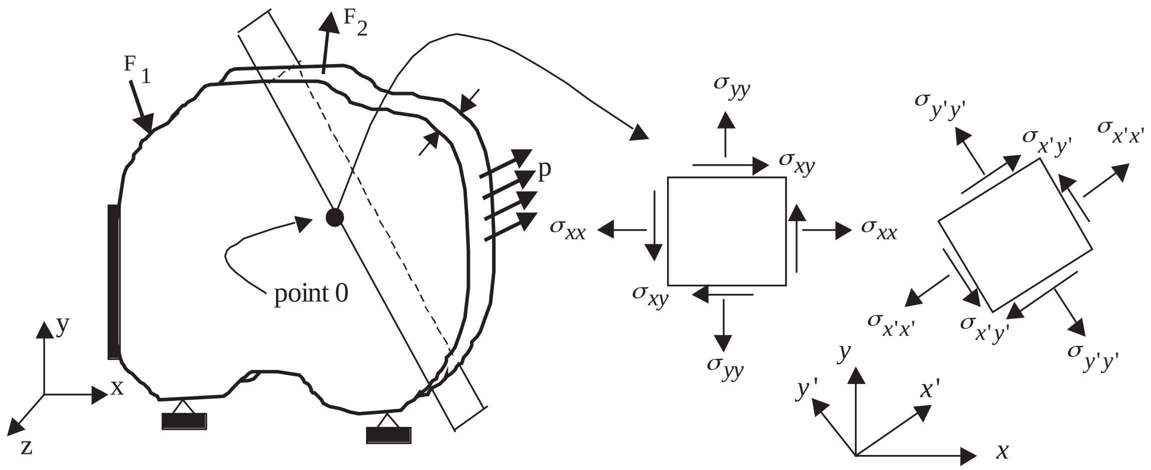


Figure 5.4: Two-Dimensional Body with Applied Loads

Now pass a cutting plane through at point "0" which has a unit normal $\mathbf{n}$ that is rotated an angle θ counter clockwise from the x-axis as shown below. Let the x'-y' coordinate system be rotated about the z-axis by the same angle θ so that x' is in the direction $\mathbf{n}$.

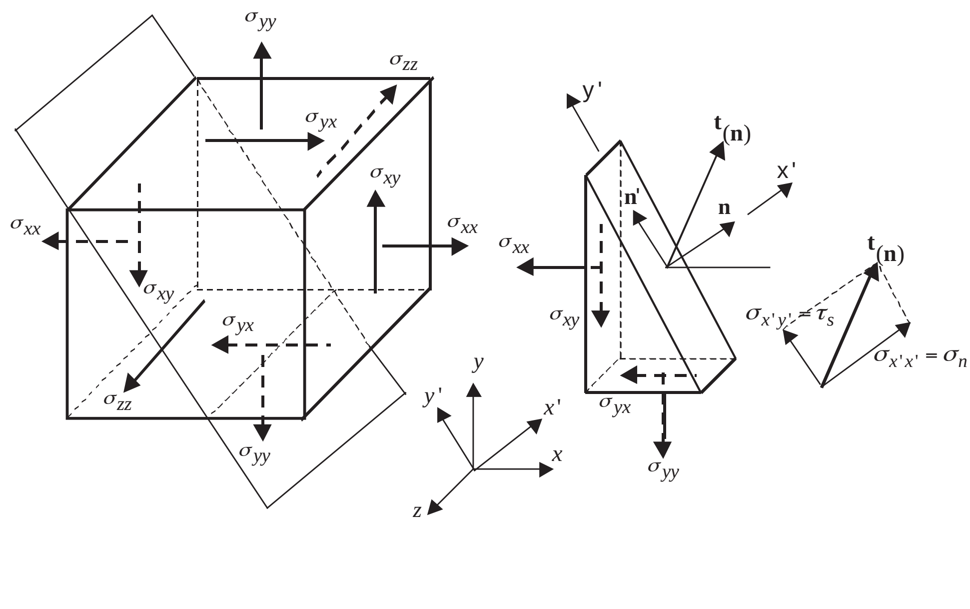


Figure 5.5: Coordinate System for Stress Transformation

For the above case the unit outer normal vector $\mathbf{n}$, is given by

$$\mathbf{n} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j} \tag{5.1}$$

Therefore, application of Cauchy's formula ($\mathbf{t}_{(\mathbf{n})} = \mathbf{n}\cdot\boldsymbol{\sigma}$) results in the following components for $\mathbf{t}_{(\mathbf{n})}$ (the stress tensor is assumed to be symmetric and therefore $\sigma_{yx} = \sigma_{xy}$):

$$\begin{aligned} \mathrm{t}_{(\mathbf{n})_x} &= \sigma_{xx}\cos\theta + \sigma_{yx}\sin\theta \\ \mathrm{t}_{(\mathbf{n})_y} &= \sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta \\ \mathrm{t}_{(\mathbf{n})_z} &= 0 \end{aligned} \tag{5.2}$$

However, these components are of no particular physical interest since they are neither normal nor parallel to the plane defined by $\mathbf{n}$. The components of $\mathbf{t}_{(\mathbf{n})}$ that are normal and parallel to the plane defined by $\mathbf{n}$ can be easily obtained through vector calculus. Note that the unit normal $\mathbf{n}$ and the x'-axis have the same direction. The component of $\mathbf{t}_{(\mathbf{n})}$ normal to the plane is thus $\sigma_{x'x'}$ and is given by the dot product of $\mathbf{t}_{(\mathbf{n})}$ with $\mathbf{n}$ (note: first substitute $\sigma_{yx} = \sigma_{xy}$ in $\mathrm{t}_{(\mathbf{n})_x}$):

$$\begin{aligned} \sigma_{x'x'} = \mathbf{t}_{(\mathbf{n})}\cdot\mathbf{n} &= ((\sigma_{xx}\cos\theta + \sigma_{xy}\sin\theta)\mathbf{i} + (\sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta)\mathbf{j})\cdot(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \\ &= \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \end{aligned} \tag{5.3}$$

where the x'-y' axes are rotated **counterclockwise** from the x-axes by the angle θ.

Since the x' axis is in the direction of $\mathbf{n}$, $\sigma_{x'x'}$ (stress normal to plane with unit normal $\mathbf{n}$) is often denoted as $\sigma_{\mathbf{n}}$. Equations 5.2 and 5.3 can also be combined by writing $\mathbf{t}_{(\mathbf{n})} = \mathbf{n}\cdot\boldsymbol{\sigma}$ and $\sigma_{\mathbf{n}} = \sigma_{x'x'} = \mathbf{t}_{(\mathbf{n})}\cdot\mathbf{n}$ so that we have the following vector and matrix result:

$$\sigma_{\mathbf{n}} = \sigma_{x'x'} = \mathbf{n}\cdot\boldsymbol{\sigma}\cdot\mathbf{n} = \overset{(1\times2)}{[\mathbf{n}]}\overset{(2\times2)}{[\boldsymbol{\sigma}]}\overset{(2\times1)}{[\mathbf{n}]} \tag{5.4}$$

In order to obtain the component of $\mathbf{t}_{(\mathbf{n})}$ parallel to the plane defined by $\mathbf{n}$, it is first convenient to define the unit normal in the y' direction, denoted $\mathbf{n}'$, as follows:

$$\mathbf{n}' = \mathbf{k}\times\mathbf{n} = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j} \tag{5.5}$$

The projection of $\mathbf{t}_{(\mathbf{n})}$ on the plane $\mathbf{n}$ is found by taking the dot product of $\mathbf{t}_{(\mathbf{n})}$ with $\mathbf{n}'$, thus yielding the shear stress component $\sigma_{x'y'}$,

$$\begin{aligned} \sigma_{x'y'} &= \mathbf{t}_{(\mathbf{n})}\cdot\mathbf{n}' \implies \\ \sigma_{x'y'} &= -(\sigma_{xx} - \sigma_{yy})\sin\theta\cos\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta) \end{aligned} \tag{5.6}$$

In the same manner, $\sigma_{y'y'}$ and $\sigma_{y'x'}$ can be determined (note: this step is not necessary):

$$\begin{aligned} \sigma_{y'y'} &= \mathbf{t}_{(\mathbf{n}')}\cdot\mathbf{n}' \implies \\ \sigma_{y'y'} &= [-(-\sigma_{xx}\sin\theta + \sigma_{xy}\cos\theta)\sin\theta + (-\sigma_{yx}\sin\theta + \sigma_{yy}\cos\theta)\cos\theta] \\ \sigma_{y'x'} &= \mathbf{t}_{(\mathbf{n}')}\cdot\mathbf{n} \implies \\ \sigma_{y'x'} &= [(-\sigma_{xx}\sin\theta + \sigma_{xy}\cos\theta)\cos\theta + (-\sigma_{yx}\sin\theta + \sigma_{yy}\cos\theta)\sin\theta] \end{aligned}$$

Equations 5.3 and 5.6 can now be used to obtain the normal stress, $\sigma_{x'x'}$, and shear stress, $\sigma_{x'y'}$, on a plane defined by the angle θ (measured CCW from x-axis), given σ_{xx}, σ_{yy} and σ_{xy}:

Stress Transformation from x-y to x'-y' Coordinates

$$\begin{aligned} \sigma_{x'x'} &= \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \\ \sigma_{x'y'} &= -(\sigma_{xx} - \sigma_{yy})\sin\theta\cos\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta) \end{aligned} \tag{5.7}$$

Although the above equations are sufficient to perform stress transformations, they are not very convenient. This is due to the fact that we are often interested in finding the planes defined by θ on which $\sigma_{x'x'}$ and $\sigma_{x'y'}$ attain their maxima because failure is often initiated on these planes due to the stresses on these planes. Mathematically, an equation for the plane of maximum (or minimum) stress can be obtained from calculus by applying the following:

$$\begin{aligned} \frac{d\sigma_{x'x'}}{d\theta} &= -(\sigma_{xx} - \sigma_{yy}) \sin 2\theta + 2\sigma_{xy} \cos 2\theta = 0 \\ \frac{d\sigma_{x'y'}}{d\theta} &= -2\sigma_{xy} \sin 2\theta - (\sigma_{xx} - \sigma_{yy}) \cos 2\theta = 0 \end{aligned} \tag{5.8}$$

Equations (5.8) can be solved for θ that maximizes/minimizes $\sigma_{x'x'}$ and $\sigma_{x'y'}$. Note that one obtains one value from each equation, i.e., θ defining the plane of maximum normal stress $\sigma_{x'x'}$ (call it θ_P), and a second θ for the maximum shear stress (call it θ_S). It can be shown that $\theta_S = \theta_P \pm \frac{\pi}{4}$. Thus, *the plane of maximum shear stress is always* $\pm 45°$ *from the plane of maximum normal stress.* These corresponding two values of θ may then be substituted into (5.7) to obtain the maximum and minimum values of $\sigma_{x'x'}$ and $\sigma_{x'y'}$. Because equations (5.8) are transcendental, closed form solutions for θ do not exist; however they can be solved numerically or graphically.

It is important to note that numerical evaluations using equations (5.7) and (5.8) must be done with the *angle θ in radians, not degrees.* Furthermore, *θ is positive counterclockwise from the x-axis* (normal right-hand rule for an x-y coordinate system).

In order to deal with the numerical difficulty associated with the nonlinear equations, Otto Mohr introduced a graphical technique that is helpful in performing stress transformations. We shall now derive this graphical technique using equations . To accomplish this, first recall the trigonometric half angle formulas:

$$\begin{aligned} \cos^2\theta &= \frac{1+\cos 2\theta}{2} \\ \sin^2\theta &= \frac{1-\cos 2\theta}{2} \\ 2\sin\theta\cos\theta &= \sin 2\theta \end{aligned} \tag{5.9}$$

Substituting (5.9) into (5.7) results in

$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\cos 2\theta + \sigma_{xy}\sin 2\theta \tag{5.10}$$

$$\sigma_{x'y'} = -\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \sigma_{xy}\cos 2\theta \tag{5.11}$$

Otto Mohr (1870) discovered that equations (5.10) and (5.11) can be combined by squaring each side of the above two equations and adding the results together to produce **the principle of Mohr's Circle** represented by the equation:

$$\left[\sigma_{xx} - \left(\frac{\sigma_{xx}+\sigma_{yy}}{2}\right)\right]^2 + \sigma_{x'y'}^2 = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2 \tag{5.12}$$

Recall that the equation of a circle in x-y space is given by

$$[x-a]^2 + [y-b]^2 = r^2 \tag{5.13}$$

where a and b are the x and y intercepts of the center of the circle, respectively, and r is the radius, as shown below:

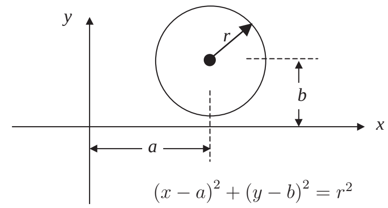


Figure 5.6: Circle Used in Mohr's Analogy

Thus by analogy of (5.12) to (5.13), we may effect the following transformation of variables:

$$
\begin{aligned}
x \rightarrow \sigma_{x'x'} & \quad & \text{(normal stress)} \\
y \rightarrow \sigma_{x'y'} & = & \text{(shear stress)} \\
a \rightarrow \frac{\sigma_{xx} + \sigma_{yy}}{2} & = & \text{(center of circle)} \\
b \rightarrow 0 & = & \\
r \rightarrow \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} & = & \text{(radius of circle)}
\end{aligned}
\tag{5.14}
$$

thereby producing the following diagram, denoted as Mohr's Circle:

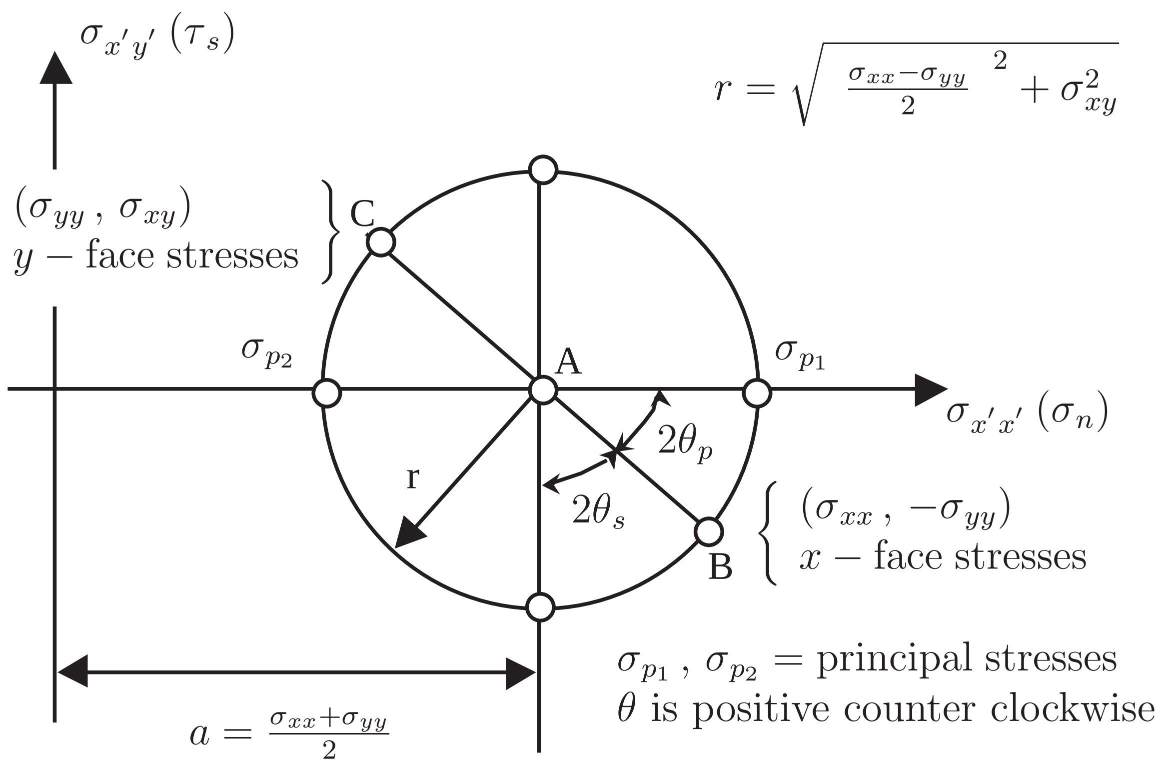


Figure 5.7: Mohr's Circle Parameters

To plot Mohr's circle given stresses σ_{xx}, σ_{yy}, σ_{xy} in the x-y coordinate system:

1. **Find the center of Mohr's circle; point** A: $\left(\frac{\sigma_{xx}+\sigma_{yy}}{2}, 0\right)$ Note: $\sigma_A = \frac{\sigma_{xx}+\sigma_{yy}}{2}$
2. **Plot stresses on x-face as point** B $(\sigma_{xx}, -\sigma_{xy})$ **and label as x-face**. Note that point B corresponds to $\theta = 0°$ in x-y plane.
3. **Plot stresses on y-face as point** C $(\sigma_{yy}, \sigma_{xy})$ **and label as y-face.** Note that point C corresponds to $\theta = 90°$ in x-y plane.
4. **Draw the circle.** We note that while the x-face ($\theta = 0°$) and the y-face ($\theta = 90°$) are separated by 90° in the real world x-y coordinates, they appear as point B and C, respectively, in Mohr's Circle and are 180° apart on Mohr's Circle. Consequently, *angles on Mohr's Circle are twice that of the real x-y world.* It is also important to note that $\theta = 0°$ *in the real world corresponds to the x-axis; however, in Mohr's Circle the point* $\theta = 0°$ *starts from the line AB* since point B represents the stresses on the x-face $\theta = 0°$). θ *is positive CCW* in both cases.
5. **Label principal stresses and maximum shear stress**. Note: $\sigma_P = \sigma_A \pm r$ and $\tau_{S_{\max}} = \pm r$. There are two principal stresses σ_{P1} and σ_{P2} (planes where shear stress is zero) and two maximum shear stresses $\tau_{S_{\max}} = \pm r$ (top and bottom of circle). Note that σ_{P1} and σ_{P2} are 180° apart on Mohr's circle, but 90° in the real world. σ_{P1} is the normal stress in the x' direction where x' is rotated by the angle θ (CCW) from the x-axis. σ_{P2} is in the y' direction.
6. **Identify the relative orientation between x-face (B) and the principal planes.** *Note that point B corresponds to the stresses on the x-face, i.e. $\theta = 0°$ in the real world.* For example, in the above $2\theta_P = \tan^{-1}\left(\frac{\sigma_{xy}}{\sigma_{xx}-\sigma_A}\right)$ CCW from the x-face (from line A-B on Mohr's Circle). Note: the writing of a specific formula is discouraged. It is far simpler (and usually less mistakes) to look at Mohr's circle and apply trigonometry to calculate the angle.
7. **Identify the relative orientation between x-face (B) and the maximum shear planes.** From Figure 5.7, $2\theta_P + 2\theta_S = 90°$. Thus $\theta_S = 45° - \theta_P$. From Figure 5.7, it easier to identify the principal shear stress planes as being $-45°$ (real world) from the principal normal stress plane (point B, bottom of circle) and $+45°$ from the principal normal stress plane (top of circle). Note that the plane of max shear stress will generally have non-zero normal stress!
8. **Draw Free Body Diagrams with the x-face, principal plane, and the two maximum shear planes.**

Example 5-1

Consider the plane stress state given by the stress tensor:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 50 & 10 \\ 10 & 20 \end{bmatrix} \text{ ksi}$$

1. Determine center of circle, point A.

$$\sigma_A = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{50 + 20}{2} = 35 \text{ ksi}$$

2. Determine point B, x-face $\rightarrow (\sigma_{xx}, -\sigma_{xy}) = (50, -10)$
3. Determine point C, y-face $\rightarrow (\sigma_{yy}, \sigma_{xy}) = (20, +10)$
4. Draw Mohr's circle:

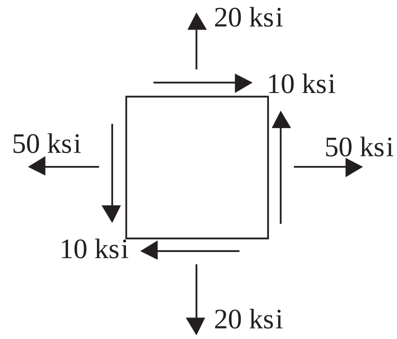


Figure 5.8:

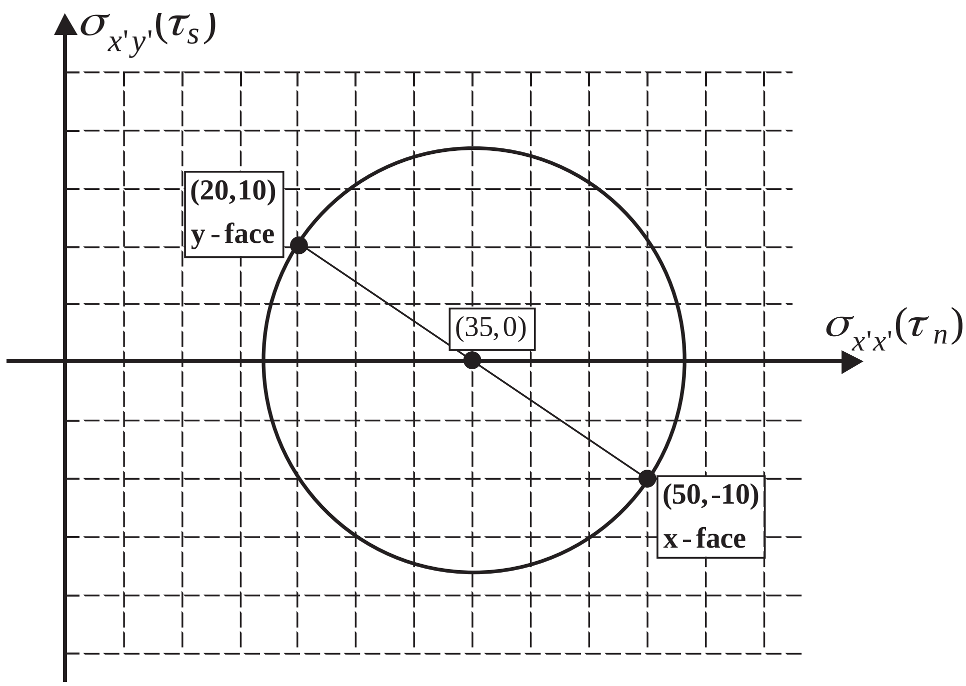


Figure 5.9:

5. Compute principal stresses and max shear stress.

$$r = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{\left(\frac{50 - 20}{2}\right)^2 + 10^2} = 18.03 \text{ ksi}$$

$$\sigma_P = \sigma_A \pm r = 35 \pm 18.03$$

$$\therefore \sigma_{P1} = 35 + 18.03 = 53.03 \text{ ksi}$$

$$\sigma_{P2} = 35 - 18.03 = 16.97 \text{ ksi}$$

$$\tau_{S_{\max}} = \pm r = \pm 18.03 \text{ ksi}$$

Note that we have two principal stresses: σ_{P1} and σ_{P2}. These are located 180° on Mohr's circle, but 90° in the real world. In relation to the stress transformation equations (5.7), σ_{P1} in the normal stress in x'-axis direction (which is oriented at an angle θ_P to the x-axis) and σ_{P2} is the normal stress in the y'-axis direction. The planes of maximum shear occur 45° (real world) from the planes of principal stress.

Plot the principal stresses and max shear stresses on Mohr's Circle:

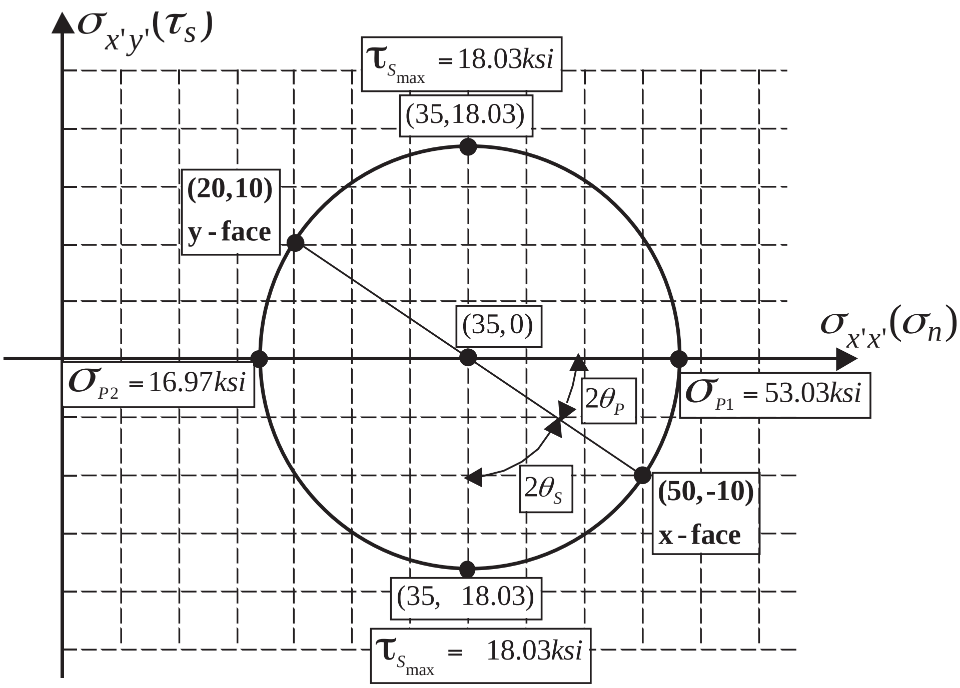


Figure 5.10:

6. Identify orientation of planes for principal stresses relative to x-face plane.

$$2\theta_P = \tan^{-1}\left(\frac{\sigma_{xy}}{\sigma_{xx} - \sigma_A}\right) = \tan^{-1}\left(\frac{10}{50-35}\right) = 33.7 \text{ deg}$$

$$\therefore \theta_P = 16.85 \text{ deg} \quad \text{CCW from } x\text{-face}$$

The plane for the second principal stress σ_{P2} is $(\theta_P + 90)$ deg CCW from x-face (real world). Note: In the above calculation, a formula was written down and used for $2\theta_P$. This is discouraged. It is far simpler (and usually less mistakes) to look at Mohr's circle and apply trigonometry to calculate the angles.

7. Identify orientation of planes for maximum shear stress (bottom of circle) relative to x-face plane.

$$2\theta_S = 90 \text{ deg} - 2\theta_P = 90 - 33.7 = 56.3 \text{ deg}$$

$$\therefore \theta_S = 28.15 \text{ deg} \quad \text{CW from the } x\text{-face}$$

OR, plane of max shear stress (bottom of circle) is 45° CW in the real world from the principal stress plane with σ_{P1}.

Note: $|\sigma_P| + |\sigma_S| = 45$ deg (*always*!)

Note that the plane of max shear stress will generally have non-zero normal stress! In this case, the plane of max shear stress has a normal stress $\sigma_n = 35$ ksi.

8. Draw three free bodies: with the x-y stresses, with the principal stresses, and with the max shear stresses.

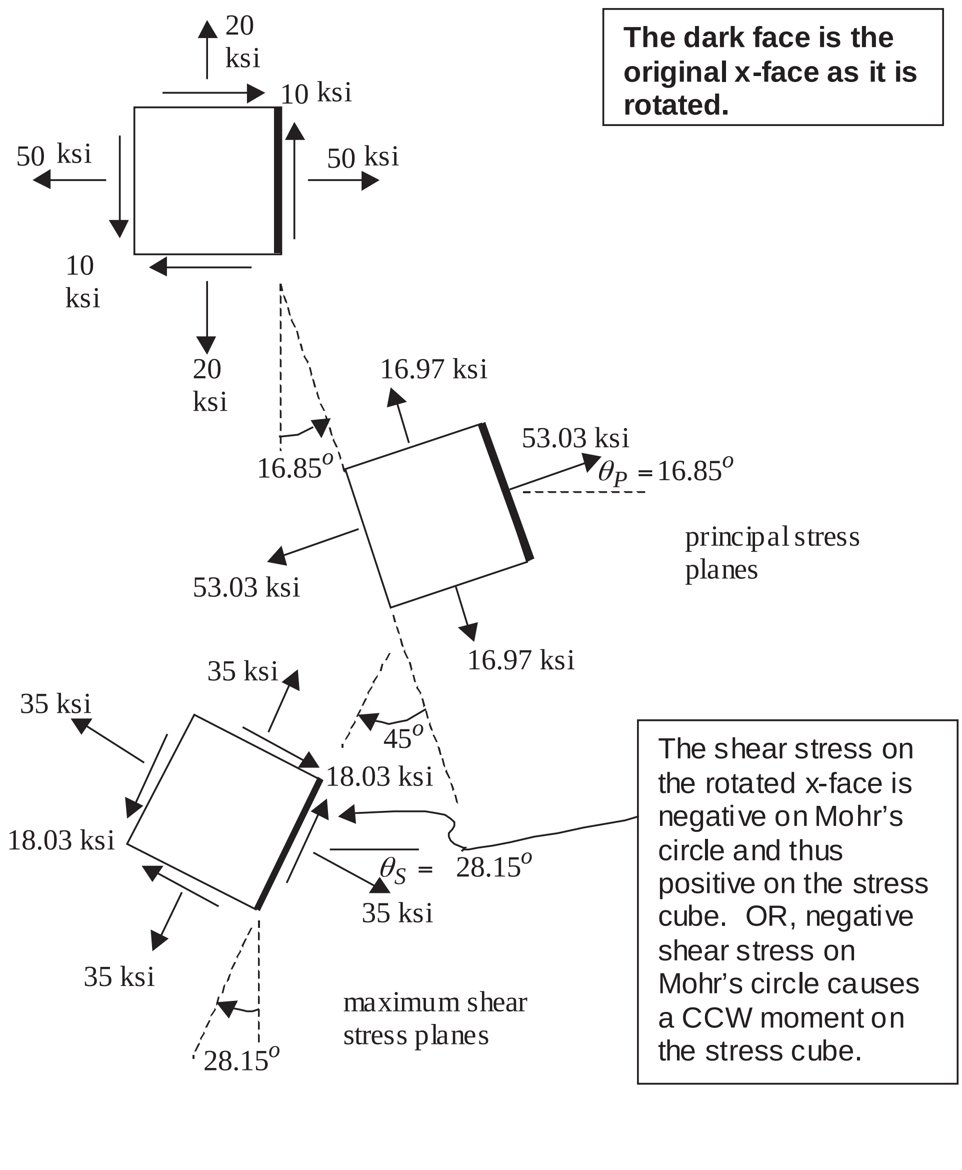


Figure 5.11:

Note the following:

- To obtain the orientation for principal stresses, the x-face has been rotated 16.85 deg CCW in the real world (33.7 deg on Mohr's circle).
- To obtain the orientation for maximum shear stresses, the x-face has been rotated 28.15 deg CW in the real world (56.3 deg on Mohr's circle), OR, equivalently a rotation of 45 deg CW in the real world from the principal stress plane.

Example 5-2

Consider the same plane stress state given in Example 5-1:

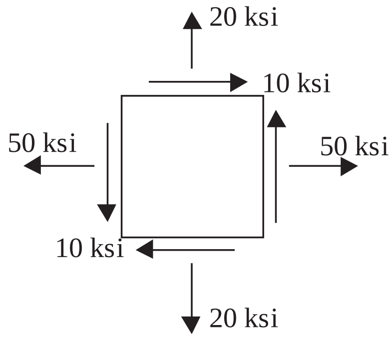


Figure 5.12:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 50 & 10 \\ 10 & 20 \end{bmatrix} \text{ ksi}$$

Values of principal stress and maximum shear stress obtain in Example 5-1 by Mohr's circle must be identical to what would be obtained by using the stress transformation equations (5.7). This is true because Mohr's circle is simply a graphical representation of equations (5.7). Use the stress transformations to determine the normal stress and shear stress for values of $\theta = 16.85°$, $106.85°$, $-28.15°$ and verify that they correspond to the principal stress and maximum shear stress values obtained in Example 5-1.

For $\theta = 16.85°$ (same as 33.7° on Mohr's circle from x-face, should equal σ_{P1})

$$\begin{aligned}
\sigma_{x'x'} &= \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \\
&= (50\text{ ksi})\cos^2 16.85° + 2(10\text{ ksi})\sin 16.85°\cos 16.85° + (20\text{ ksi})\sin^2 16.85° = 53.03\text{ ksi} \\
\sigma_{x'y'} &= -(\sigma_{xx} - \sigma_{yy})\sin\theta\cos\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta) \\
&= -(50\text{ ksi} - 20\text{ ksi})\sin 16.85°\cos 16.85° + 10\text{ ksi}(\cos^2 16.85° - \sin^2 16.85°) = 0\text{ ksi}
\end{aligned}$$

For $\theta = 106.85°$ (same as 213.7° on Mohr's circle from x-face, should equal σ_{P2})

$$\begin{aligned}
\sigma_{x'x'} &= \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \\
&= (50\text{ ksi})\cos^2 106.85° + 2(10\text{ ksi})\sin 106.85°\cos 106.85° + (20\text{ ksi})\sin^2 106.85° = 16.97\text{ ksi} \\
\sigma_{x'y'} &= -(\sigma_{xx} - \sigma_{yy})\sin\theta\cos\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta) \\
&= -(50\text{ ksi} - 20\text{ ksi})\sin 106.85°\cos 106.85° + 10\text{ ksi}(\cos^2 106.85° - \sin^2 106.85°) = 0\text{ ksi}
\end{aligned}$$

For $\theta = -28.15°$ (same as $-56.3°$ on Mohr's circle from x-face, should equal $\tau_{\max}$ bottom of circle)

$$\begin{aligned}
\sigma_{x'x'} &= (50\text{ ksi})\cos^2(-28.15°) + 2(10\text{ ksi})\sin(-28.15°)\cos(-28.15°) + (20\text{ ksi})\sin^2(-28.15°) = 35\text{ ksi} \\
\sigma_{x'y'} &= -(50\text{ ksi} - 20\text{ ksi})\sin(-28.15°)\cos(-28.15°) + 10\text{ ksi}(\cos^2(-28.15°) - \sin^2(-28.15°)) = 18.03\text{ ksi}
\end{aligned}$$

For $\theta = 40°$ (same as 80° on Mohr's circle from x-face)

$\sigma_{x'x'} = 47.45$ ksi, $\sigma_{x'y'} = -13.04$ ksi (real world values). From, Mohr's circle in Example 5-1, one obtains the same result. ⇐ **Student, you should verify this**! Note that this corresponds to a point 80 degrees CCW from the x-face on Mohr's circle (or 46.3 degrees above the σ_n axis on Mohr's circle (80, −33.7)). Why is the sign negative on the value of $\sigma_{x'y'}$ calculated from the stress transformation equation? Remember that when you plot the x-face stress, a positive σ_{xy} (real world) is plotted as $-\sigma_{xy}$ (point B on Mohr's circle). Thus, a positive value of shear stress on Mohr's circle as we have for this case ($\theta = 80°$ CCW on Mohr's circle) translates to a negative shear stress in the real world.

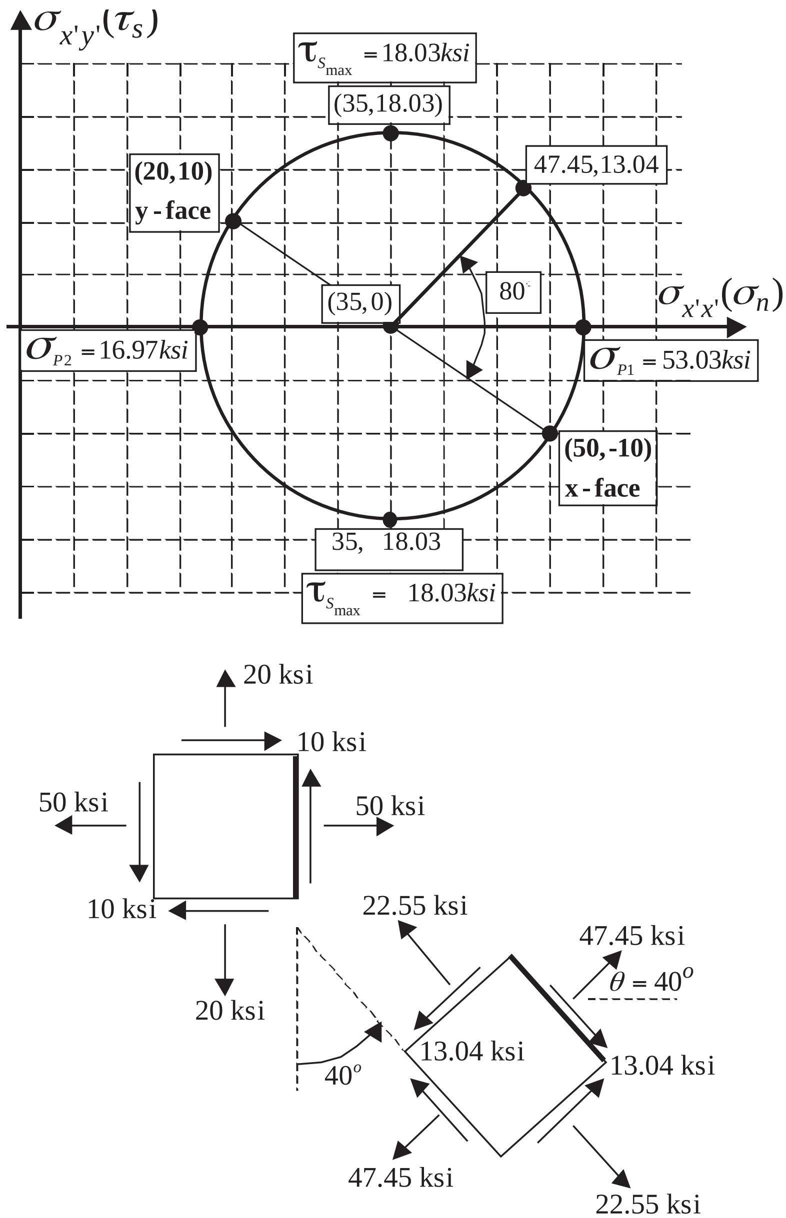


Figure 5.13:

Example 5-3

Consider the plane stress state given by the stress tensor:

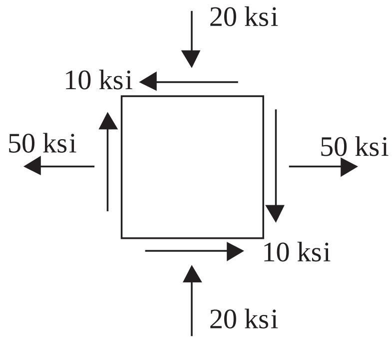


Figure 5.14:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 50 & -10 \\ -10 & -20 \end{bmatrix} \text{ ksi}$$

1. Determine center of circle, point A.

$$\sigma_A = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{50 + (-20)}{2} = 15 \text{ ksi}$$

2. Determine point B, x-face $\rightarrow (\, \sigma_{xx}, -\sigma_{xy} \,) = (\, 50, -(-10) \,) = (\, 50, 10 \,)$
3. Determine point C, y-face $\rightarrow (\, \sigma_{yy}, \sigma_{xy} \,) = (\, -20, -10 \,)$
4. Draw Mohr's circle:
5. Compute principal stresses and max shear stress.

$$r = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{\left(\frac{50 - (-20)}{2}\right)^2 + (-10)^2} = 36.4 \text{ ksi}$$

$$\sigma_P = \sigma_A \pm r = 15 \pm 36.4$$

$$\therefore \sigma_{P1} = 15 + 36.4 = 51.4 \text{ ksi}$$

$$\sigma_{P2} = 15 - 36.4 = -21.4 \text{ ksi}$$

$$\tau_{S_{\max}} = \pm r = \pm 36.4 \text{ ksi}$$

Note that we have two principal stresses: σ_{P1} and σ_{P2}. These are located 180° on Mohr's circle, but 90° in the real world. In relation to the stress transformation equations (5.7), σ_{P1} in the normal stress in x'-axis direction (which is oriented at an angle θ_P to the x-axis) and σ_{P2} is the normal stress in the y'-axis direction. The planes of maximum shear occur 45° (real world) from the planes of principal stress.

Plot the principal stresses and max shear stresses on Mohr's Circle:

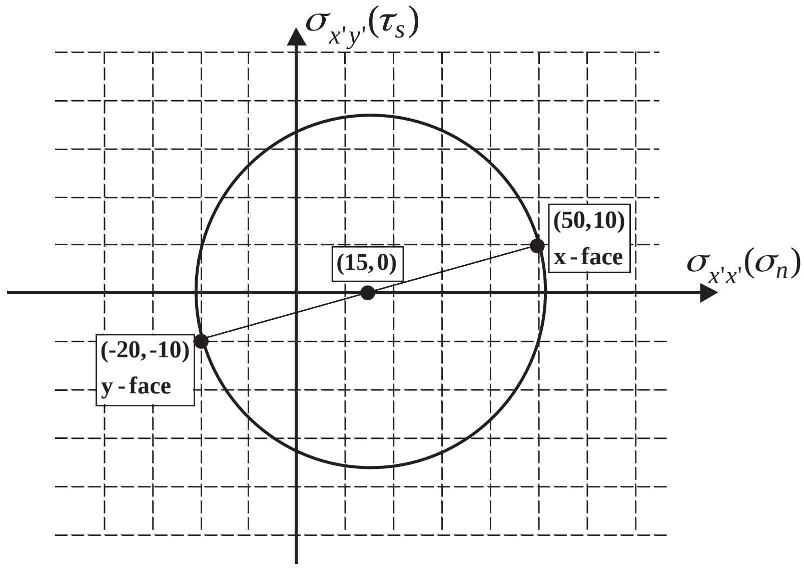


Figure 5.15:

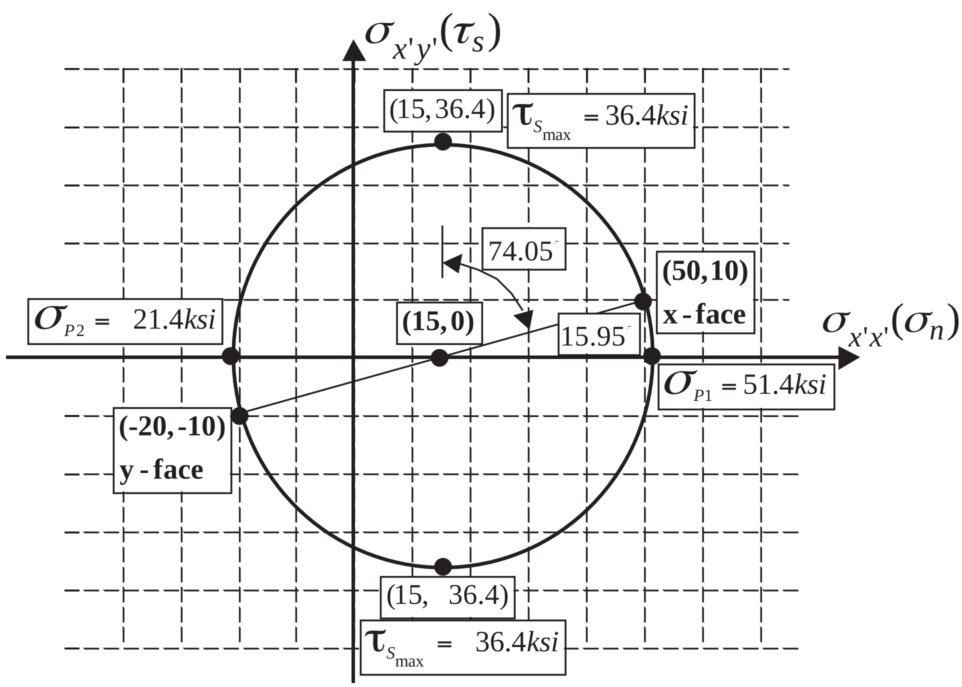


Figure 5.16:

6. Identify orientation of planes for principal stresses relative to x-face plane.

$$2\theta_P = \tan^{-1}\left(\frac{10}{35}\right) = 15.95 \text{ deg}$$

$$\therefore \theta_P = 7.97 \text{ deg} \quad \text{CW from } x\text{-face}$$

The plane for the second principal stress σ_{P2} is $(\theta_P + 90)$deg CCW from x-face (real world).

7. Identify orientation of planes for maximum shear stress (bottom of circle) relative to x-face plane.

$$2\theta_S = 90 \text{ deg} - 2\theta_P = 90 - 15.95 = 74.05 \text{ deg}$$

$$\therefore \theta_S = 37.03 \text{ deg} \quad \text{CCW from the } x\text{-face}$$

or, plane of max shear stress (top of circle) is 45° CCW from plane with σ_{P1}. *Note that the plane of max shear stress will generally have non-zero normal stress*! In this case, the plane of max shear stress has a normal stress $\sigma_n = 15$ ksi.

8. Draw three free bodies: with the x-y stresses, with the principal stresses, and with the max shear stresses.

Note the following:

- To obtain the orientation for principal stresses, the x-face has been rotated 7.97 deg CW in the real world (15.95 deg on Mohr's circle).
- To obtain the orientation for maximum shear stresses, the x-face has been rotated 37.03 deg CCW in the real world (74.05 deg on Mohr's circle), or, equivalently a rotation of 45 deg CCW in the real world from the principal stress plane.

5.2 Generalized Plane Stress and Principal Stresses

In the x-y plane, any set of stresses $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ can be represented by a Mohr's Circle as was shown previously and as shown in Figure 5.18 below (the dark circle). Corresponding to these x-y stresses, there are two principal stresses $(\sigma_{P1}, \sigma_{P2})$ which also define *the same Mohr's Circle* with coordinates of $(\sigma_{P1}, 0)$ and $(\sigma_{P2}, 0)$. Thus, the circle is (or can be) defined by any two principal stresses as they form the diameter of the circle. It follows that the Mohr's circle for the x-z plane would also be formed by the principal stresses in the x-z plane, in this case σ_{P1} and σ_{P3}. Similarly, Mohr's circle in the y-z plane is defined by σ_{P2} and σ_{P3}. We can draw all three of these circles on one diagram as shown below.

If the value of the third principal stress is less than the second, or greater than the first, we obtain a larger circle than the circle obtain for the x-y plane and hence the maximum shear stress is also larger. For the case in the figure above, even if the out-of-plane normal stress $\sigma_{zz} = 0$, one obtains a larger maximum shear stress than was obtained from the x-y plane.

For a general stress tensor (3-D), it can be shown that the principal stresses are defined by the following eigenvalue problem:

$$|[\boldsymbol{\sigma}] - \sigma_P [\mathbf{I}]| = 0$$

$$\begin{vmatrix} (\sigma_{xx} - \sigma_P) & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & (\sigma_{yy} - \sigma_P) & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & (\sigma_{zz} - \sigma_P) \end{vmatrix} = 0$$

After the 9 components of stress $(\sigma_{xx}, \sigma_{xy}, \dots, \sigma_{zz})$ are substituted into the above, the determinant can be expanded to obtain a cubic polynomial in σ_P which will yield the three principal stress values σ_{P1}, σ_{P2}, σ_{P3}. For 2-D (x-y plane), the determinant reduces to $\begin{vmatrix} (\sigma_{xx} - \sigma_P) & \sigma_{xy} \\ \sigma_{yx} & (\sigma_{yy} - \sigma_P) \end{vmatrix} = 0$ which results in two principal stress values σ_{P1}, σ_{P2}.

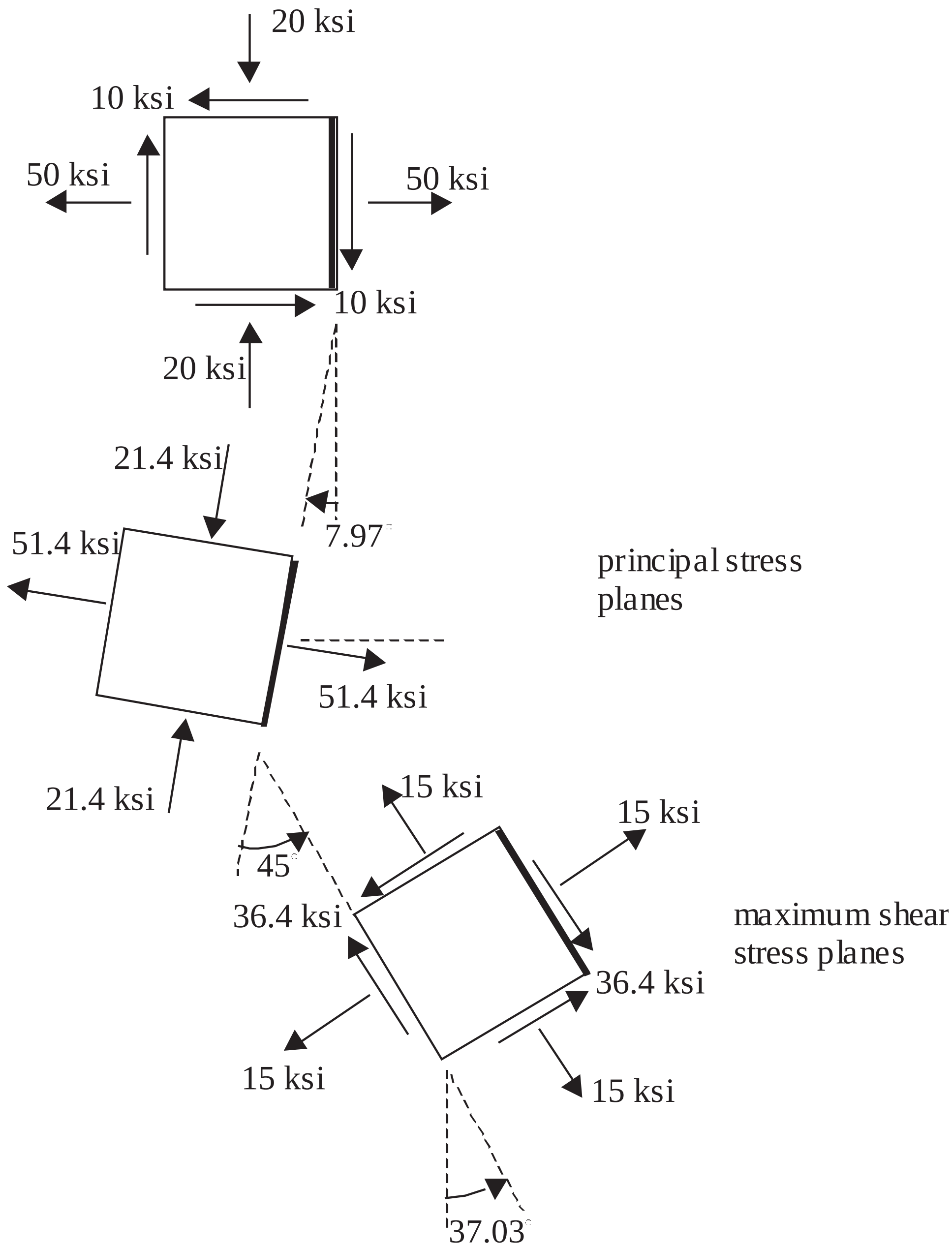


Figure 5.17:

Example 5-4

Same as Example 5-1 except a z component of stress is added to make the problem a generalized plane stress problem. The generalized plane stress state is given by the stress tensor:

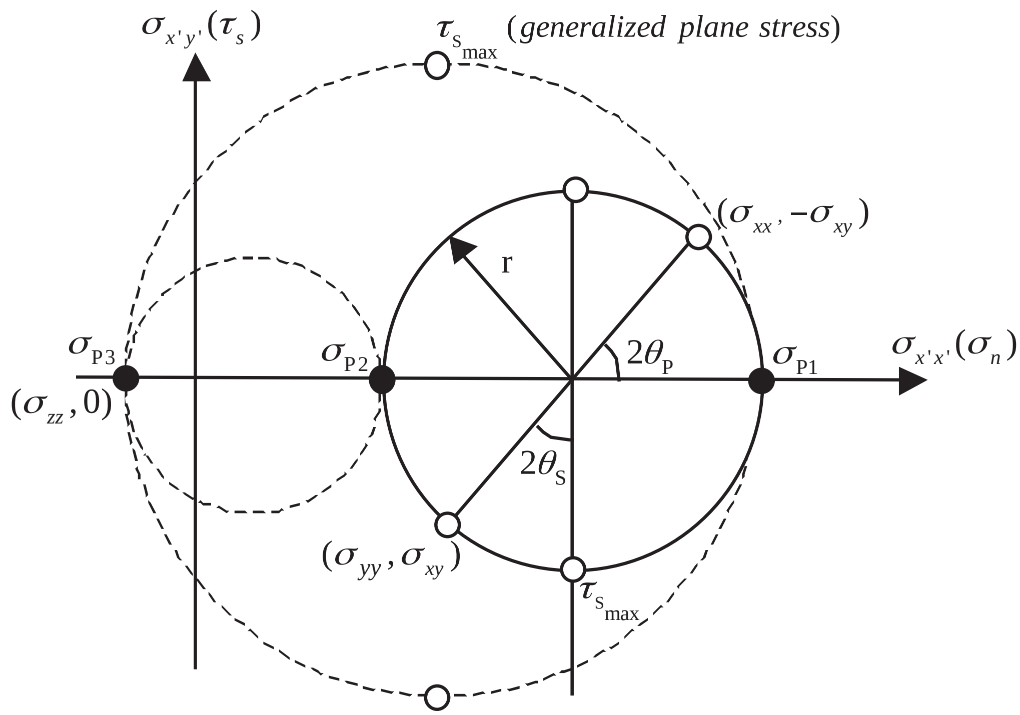


Figure 5.18: Mohr's Circle for Generalized Plane Stress (plane stress in x-y plane)

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 50 & 10 & 0 \\ 10 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix} \text{ ksi}$$

Steps 1-7 will be identical to Example 5-1 for the x-y plane (since both examples have exactly the same stress components in the x-y plane). Hence, for the x-y plane we obtain Mohr's circle:

To complete the solution we note that there are no shear stresses in the z plane and hence σ_{zz} is automatically a principal stress. Thus, $\sigma_{P3} = \sigma_{zz} = -10$. Adding the third principal stress to Mohr's circle, we obtain:

Hence, the principal stresses are $\boldsymbol{\sigma}_P = -10, 16.97, 53.03$ ksi. The maximum shear stress is the radius of the largest Mohr's circle. Thus, $\tau_{S_{\max}} = \frac{53.03+10}{2} = 31.52$ ksi. In order to "change" circles, we need to be in the principal orientation, because this is the same on all three circles. The sketches below illustrate this "changing" process. Note that when changing views, there is NOT a change in the stresses, only in the way they are viewed. If we had rotated 45° off the principal plane $P_1 - P_2$, we would have still been on the smaller circle and therefore could never attain the maximum shear stress.

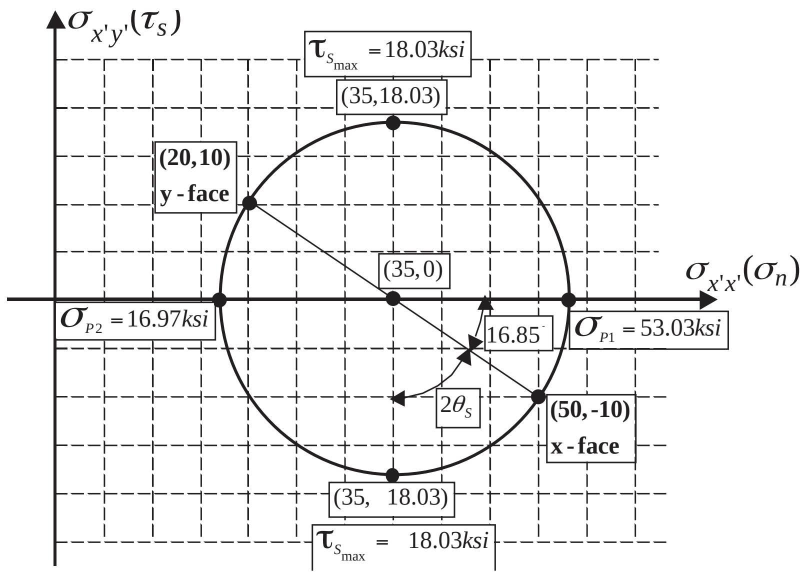


Figure 5.19:

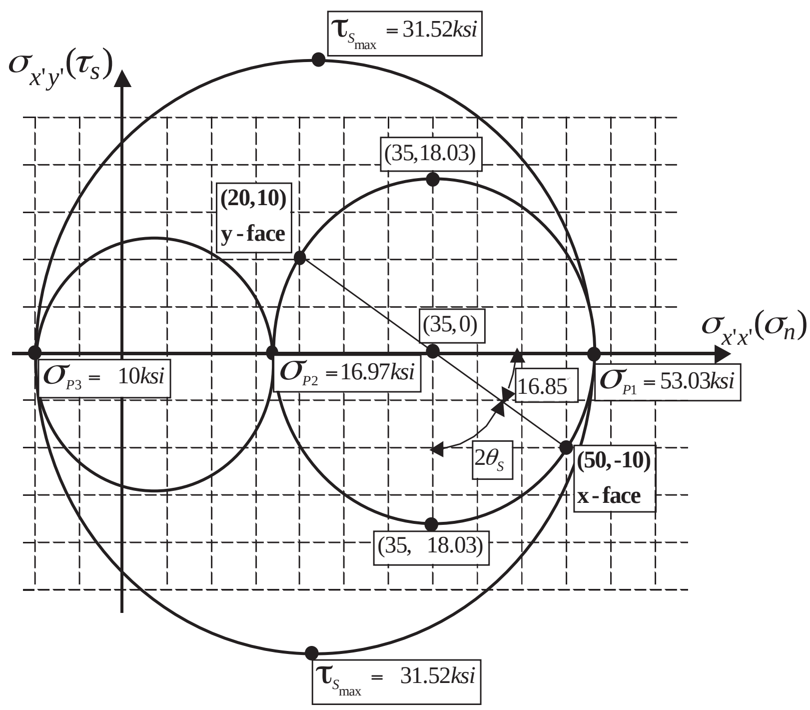


Figure 5.20:

Example 5-5

Given the stress state below, use Mohr's circle to find the principal planes and maximum shear

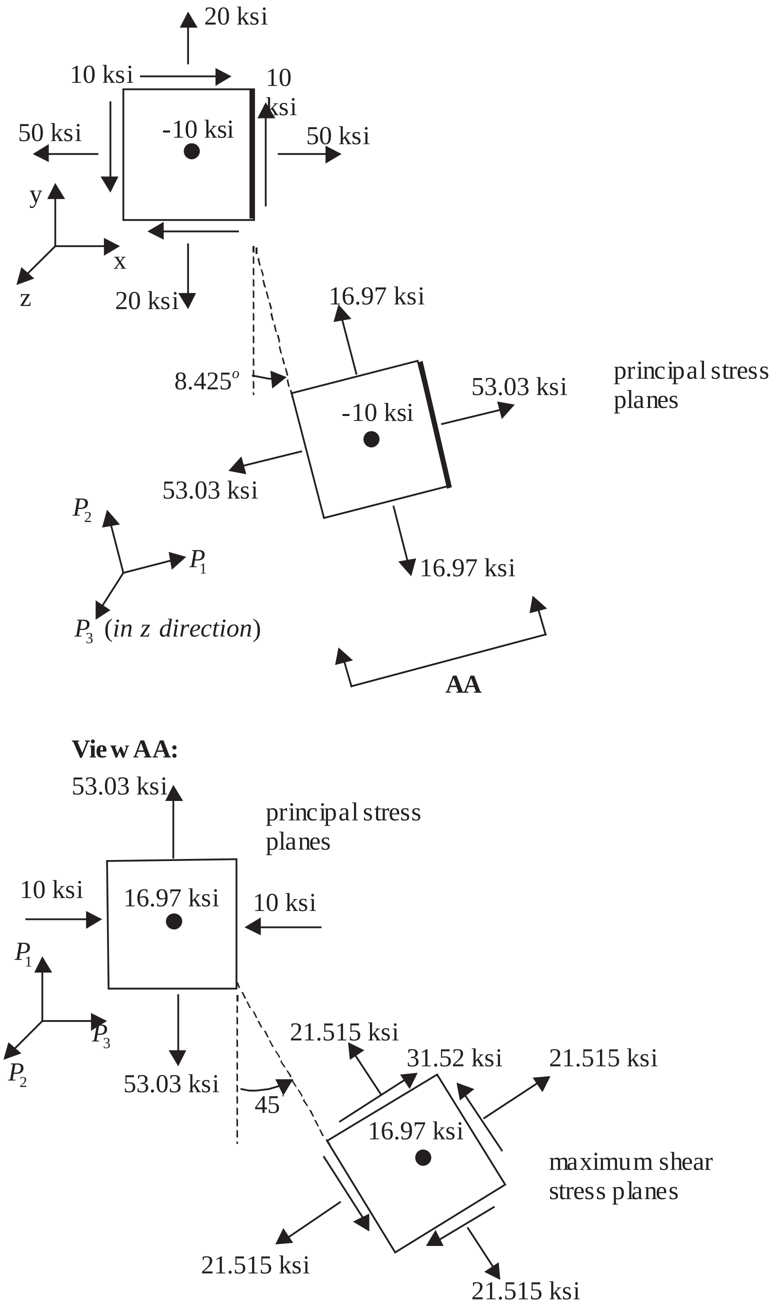


Figure 5.21:

stresses.

Uniaxial Stress

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

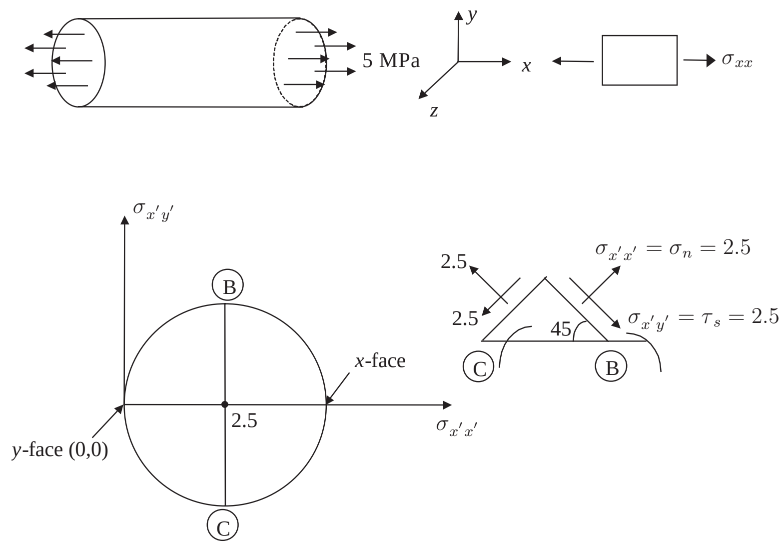


Figure 5.22:

- Maximum shear traction at 45° from x-axis
- Maximum normal traction along x-axis

Example 5-6

Given: The state of stress at each point in a bar can be represented by the stress tensor $[\boldsymbol{\sigma}]$:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 10\ \frac{\text{kN}}{\text{m}^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

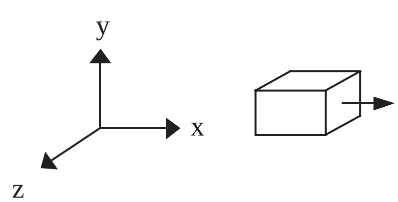


Figure 5.23:

Required:

a) Construct Mohr's Circle for the given stress tensor.

b) What are the traction vectors experienced each face of the cube?

c) What are the normal and shear components of these traction vectors?

d) What is the maximum shear stress experienced by the bar?

e) What is the orientation of the surface that experiences this maximum shear stress?

f) What is the maximum normal stress experienced by the bar?

Solution

a) Use the steps mentioned in the above text.

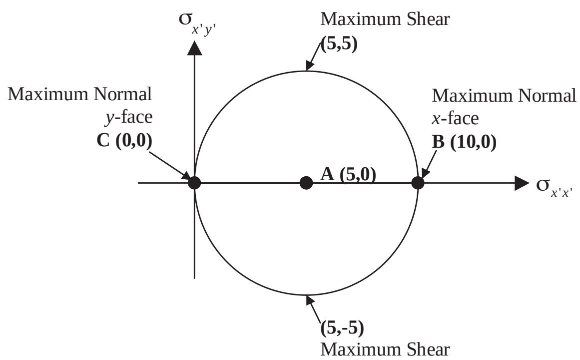


Figure 5.24:

b) $\mathbf{t_i} = 10\mathbf{i}$
$\mathbf{t_j} = \mathbf{0}$
$\mathbf{t_k} = \mathbf{0}$

c) $\sigma_{xx} = 10$
$\sigma_{xy} = \sigma_{yz} = \sigma_{zz} = \sigma_{yy} = 0$

d) $5\ \frac{\text{kN}}{\text{m}^2}$ at the points $(5, 5)$ and $(5, -5)$

e) The surface is tilted at 45° to the x-axis (90° in Mohr space is 45° in real space).

f) $10\ \frac{\text{kN}}{\text{m}^2}$ at the points $(10, 0)$ and $(0, 0)$

Example 5-7

Given: 2-D Stress State – Ignore the z components of stress.

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 0 \\ 2 & 6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

Required:

a) Construct Mohr's Circle for the given stress tensor.

b) What is the orientation of the surface when the maximum shear stress is obtained?

c) What is the maximum shear stress experienced by the bar?

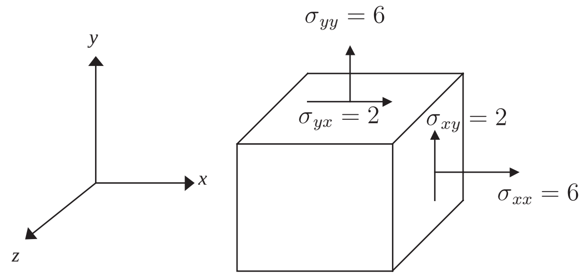


Figure 5.25:

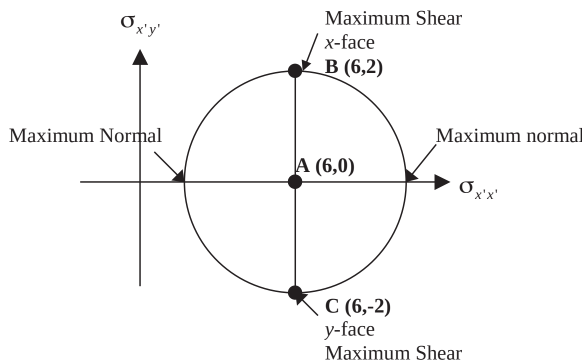


Figure 5.26:

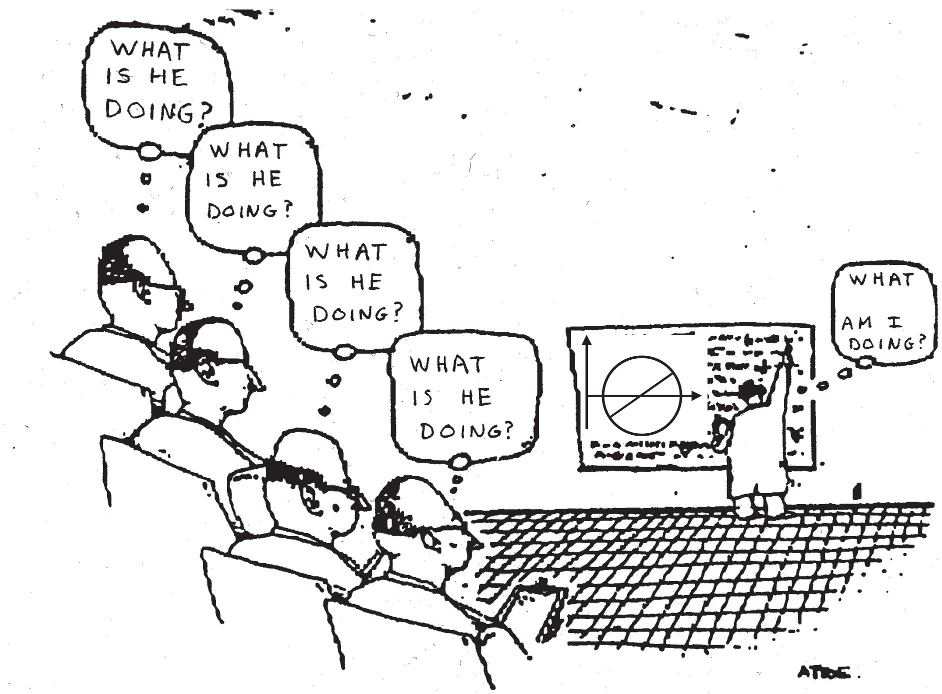
Solution

a)

b) 0°

c) 2 MPa at points $(6, 2)$ and $(6, -2)$

Deep Thought



5.3 Questions

5.1 Describe in your own words the meaning of principal stress.

5.2 What is the angle between the planes of principal stress and maximum shear stress in a body? Why is it that way?

5.3 What is the main point of Mohr's circle?

5.4 What does the term $\mathbf{t}_{(\mathbf{n})}$ in Caucy's formula physically represent?

5.5 When considering Mohr's circle, explain physically the stress state defined by the point on Mohr's Circle that lies on the horizontal axis.

5.4 Problems

5.6 *GIVEN*: A material point is known to be in a state of generalized plane stress as given by:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 4 & -2 & 0 \\ -2 & -5 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{ MPa}$$

REQUIRED: Draw all important sketches, showing planes of maximum shear as well as principal planes.

5.7 *GIVEN*: The stress tensor as follows:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ ksi}$$

REQUIRED:

(a) compute the three principal stresses and maximum shear stress;

(b) indicate the maximum shear direction.

5.8 The state of stress at a certain point of a body is given by

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

Determine on which planes:

i) principal stresses occur

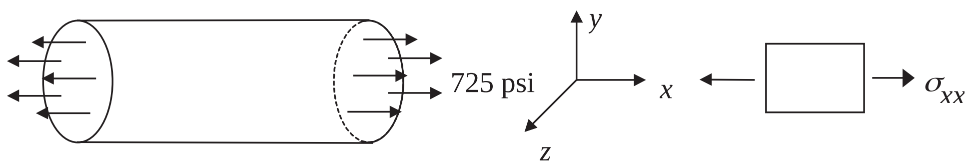
ii) shearing stress is greatest.

5.9

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ psi}$$

Find maximum shear and normal stresses.

5.10 (a) What is a principal plane?



Problem 5.9

(b) Are the following stress tensors generalized plane stress cases? Answer in yes/no.

$$\text{i)} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & -5 \end{bmatrix} \qquad \text{ii)} \begin{bmatrix} 4 & 2 & 0 \\ 2 & -3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\text{iii)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 6 \end{bmatrix} \qquad \text{iv)} \begin{bmatrix} -2 & 0 & 2 \\ 0 & 3 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

5.11 *GIVEN*:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{ ksi}$$

(a) Write out the three principal stresses and compute the maximum shear stress.

(b) What is the maximum shear direction and on what plane is it acting in?

5.12 *GIVEN*:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} -5 & -5 \\ -5 & 5 \end{bmatrix} \text{ MPa}$$

(a) Draw the corresponding stress cube.

(b) Construct the three Mohr's circles for the given stress state.

(c) Compute the two principal stresses and maximum shear stress.

(d) Using the formula given below, determine the normal stress, which acts upon a surface corresponding to a 30° *clockwise* rotation of the x-face.

$$\sigma_n = \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\cos 2\theta + \sigma_{xy}\sin 2\theta + \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right)$$

5.13 *GIVEN*: A material in the following state of plane stress:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} -2500 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ ksi}$$

REQUIRED: Draw all important sketches, showing planes of maximum shear stress as well as principal planes.

5.14 Using the following 2-D stress state at a point in a body,

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 50 & 0 & 0 \\ 0 & -50 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

Find:

a) $\mathbf{t_n}$ for $\mathbf{n} = \mathbf{i}$, $\mathbf{n} = \mathbf{j}$, and $\mathbf{n} = \frac{1}{2}\left(\sqrt{3}\mathbf{i} + \mathbf{j}\right)$

b) If the coordinate axes were rotated 45 degrees clockwise, to x'-y', solve for the new values of $\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{x'y'}$. Use Cauchy's Formula.

c) Construct the three Mohr's circles for the given stress state.

5.15 The state of stress at a certain point of a body is given by

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 3 & -1 & 4 \\ -1 & 2 & 0 \\ 4 & 0 & -2 \end{bmatrix} \text{ ksi}$$

i) Find the traction vector on the plane whose normal is in the direction $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

ii) Find the traction vector on the plane whose normal is in the direction $4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

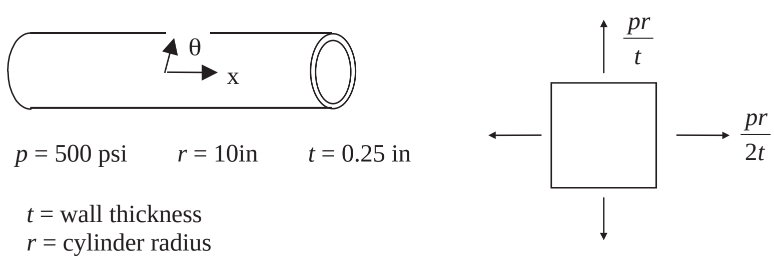
iii) Determine the normal and shearing components of the traction vector on the above planes.

5.16 For a stress state defined by: $\sigma_{xx} = 10$ MPa, $\sigma_{xy} = \sigma_{yx} = 40$ MPa, $\sigma_{yy} = 50$ MPa. Define:

a) the value of the Principal Stress

b) the maximum Shear Stress and corresponding Normal Stress

5.17 *GIVEN*: A thin-walled cylindrical pressure vessel is subjected to internal pressure, p



Problem 5.17

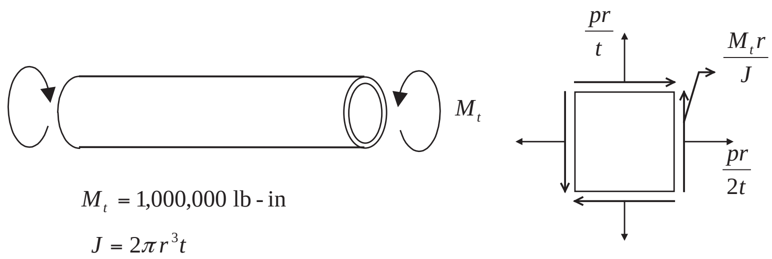
It can It can be shown that the stress state is given by a hoop (circumferential) stress $\sigma_{\theta\theta} = \frac{pr}{t}$ and an axial stress (along the axis of pressure vessel) of $\sigma_{xx} = \frac{pr}{2t}$:

REQUIRED: Draw all important sketches, showing planes of maximum shear as well as principal planes.

5.18 *GIVEN*: Suppose that the pressure vessel of 5.17 is also subjected to an end torque, M_t.

The stress state be shown by:

REQURIED: Draw all important sketches, showing planes of maximum shear as well as principal planes.



Problem 5.18

5.19 *GIVEN*: The following stress tensor in Cartesian coordinates

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 5 & -2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{ MPa}$$

REQUIRED:

a) Sketch the stress cube representation
b) Construct the three Mohr's circles
c) Determine the traction vectors: $\mathbf{t}_{(\mathbf{i})}$, $\mathbf{t}_{(\mathbf{j})}$, $\mathbf{t}_{(\mathbf{k})}$
d) Make a sketch of $\mathbf{t}_{(\mathbf{i})}$, $\mathbf{t}_{(\mathbf{j})}$, $\mathbf{t}_{(\mathbf{k})}$ acting on the appropriate plane(s)

5.20 *GIVEN*: A material point is known to be in a state of generalized plane stress as given by:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} -1 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \text{ ksi}$$

REQUIRED:

1) Construct the three Mohr's circles.
2) Find the three principal stresses and the maximum shear stress.
3) Draw all important sketches, showing planes of maximum shear as well as principal planes with the associated coordinate rotations labeled properly.

5.21 *GIVEN*: The state of stress at a certain point on a body is given by:

$$\text{a)} \qquad [\boldsymbol{\sigma}] = \begin{bmatrix} 4 & 2 & 0 \\ 2 & -3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\text{b)} \qquad [\boldsymbol{\sigma}] = \begin{bmatrix} -2 & 0 & 2 \\ 0 & 3 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

REQUIRED: Determine for both a) and b) on which of the three **coordinate planes**:

1) Normal stress is greatest
2) Shear stress is greatest

5.22 Use Cauchy's Formula, $\mathbf{t_n} = \boldsymbol{\sigma} \cdot \mathbf{n}$, for the following stress tensor

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 20 & 5 & 0 \\ 5 & -6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ ksi} \quad \mathbf{n} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j} + 0\mathbf{k}$$

REQUIRED:

a) The plane at which the shear stress equals zero, i.e., the angle in the unit normal which describes the plane for the following stress tensor.

b) The normal stress on the face described by the plane found in part a). What is this normal stress called?

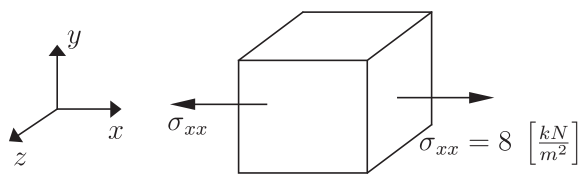
c) At what angle is the plane of maximum shear stress from the principal stress plane.

d) What is the value of the maximum shear stress for the stress tensor given above.

5.23 *GIVEN*: The state of stress at each point in a bar can be represented by the stress tensor $[\boldsymbol{\sigma}]$:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 8\,\frac{\text{kN}}{\text{m}^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

REQUIRED:



Problem 5.23

a) What are the traction vectors experienced by faces in other orientations?

b) What are the normal and shear components of these traction vectors?

c) What is the maximum shear stress experienced by the bar?

d) What is the orientation of the surface that experiences this maximum shear stress?

e) What is the maximum normal stress experienced by the bar?

5.24 *GIVEN*:

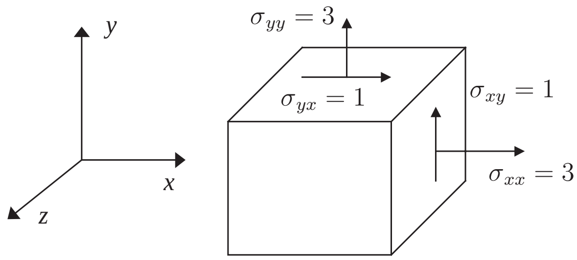
$$[\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

REQUIRED:

a) Construct Mohr's circle for the given stress tensor.

b) What is the orientation of the surface when the maximum shear stress is obtained?

c) What is the maximum shear stress experienced by the bar?



Problem 5.24

5.25 The state of stress at a certain point of a body is given by

$$[\boldsymbol{\sigma}] = \begin{bmatrix} -1 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \text{ MPa}$$

a) Calculate the values of the corresponding principal stresses.
b) Indicate the magnitude of the maximum shear stress in the x-y plane.
c) Draw the x-y plane including 1) principal stresses and 2) max shear stress. In each case, indicate the magnitude of *all* stresses and the angle the element is rotated relative to the original x-face.
d) Determine the magnitude of the maximum shear stress considering all planes.
e) If the material's failure criteria is given by a 6 MPa normal stress and 3 MPa shear stress. Indicate if the material will fail under the applied load and if so, whether it is due to shear or normal stress.

5.26 Repeat problem 5.25 for:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

Remember to consider the problem as a case of Generalized Plane Stress.

5.27 *GIVEN*: stress tensors as follows

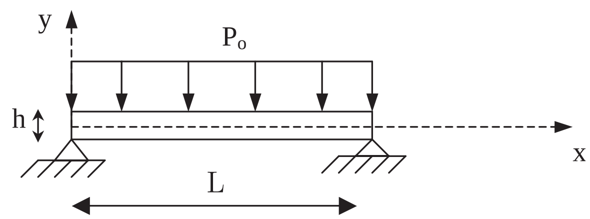
$$(1) \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2) \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad (3) \begin{bmatrix} 3 & -3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

REQUIRED:

(a) sketch the stress cube aligned with the coordinate axes (Cartesian coordinate system) and the values of all shear and normal stresses on its surfaces;
(b) construct the three Mohr's circles for the stress state;
(c) sketch a stress cube oriented so that it is aligned with the principal axes and indicate its angle relative to the x-face and the principal stresses which act upon it;
(d) determine the maximum shear and normal stresses experienced by the material which is subjected to this stress state and for each of these stresses, indicate the orientation of the surface which experiences this stress;

(e) determine the shear and normal stresses that act upon a surface which corresponding to a 30° counter-clockwise rotation of the x-face about z-axis.

5.28 For the given simply supported beam as pictured below, with length $L = 10$ m, thickness $h = 0.1$ m, cross sectional area $A = 0.01$ m^2 and a distributed load $P_o = 10$ $\frac{\text{kN}}{\text{m}}$, a stress analysis indicates a stress of $\sigma_{xx} = -2$ MPa, $\sigma_{xy} = -1$ MPa, $\sigma_{yy} = 4$ MPa at $x = 5$ m, $y = 0.05$ m. Repeat the stress analysis described in problem 5.27.



Problem 5.28

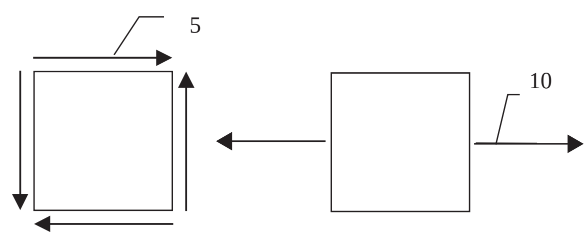
5.29 Derive an equation for $\sigma_{y'y'}$ by hand.

5.30 Using Scientific Workplace:

a) evaluate $\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{x'y'}$, $\sigma_{y'x'}$

b) verify that $\sigma_{x'y'} = \sigma_{y'x'}$ (assume $\sigma_{xy} = \sigma_{yx}$)

c) Graph $\sigma_{x'x'}(\theta)$, $\sigma_{x'y'}(\theta)$ for $0 < \theta < 2\pi$ for the two cases below and discuss your findings.



Problem 5.30

5.31 *GIVEN*: *REQUIRED*:

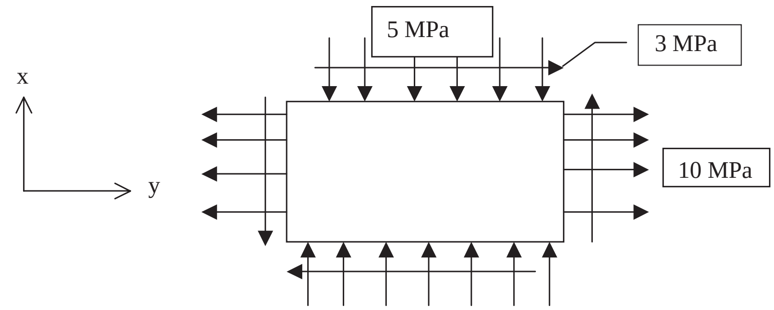
a) Find the stress tensor, $[\boldsymbol{\sigma}]$.

b) Find the principal stresses.

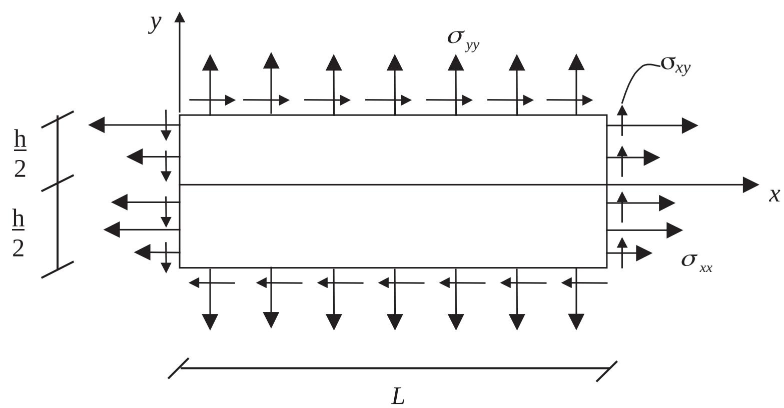
c) Find the principal direction.

5.32 *GIVEN*: A thin plate is loaded on its four faces so that it develops the following stress state.

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 7\left(x^2y - \frac{2}{3}y^2\right) & -7xy^2 & 0 \\ -7xy^2 & \frac{7}{3}y^3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$



Problem 5.31



Problem 5.32

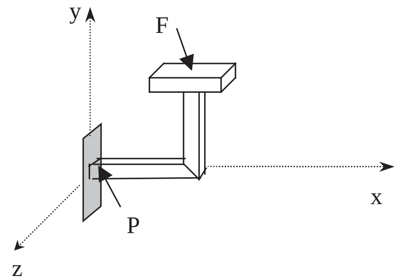
REQUIRED:

1) Verify that the conservation of angular momentum and linear momentum are satisfied under static conditions, in the absence of body forces, at *every* point (x, y) on the plate.
2) Find the components of the traction vector applied on the faces: $x = 0$, $x = L$, $y = \frac{h}{2}$, $y = -\frac{h}{2}$

5.33 For the following structure and the applied load **F**, the stress state in a rectangular Cartesian coordinate system at point P is given by the specified stress tensor $[\boldsymbol{\sigma}]$.

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 5 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{ MPa}$$

a) Sketch the stress cube representation. Label all the components, planes and points of interest.
b) Determine the values of the Principal Stresses.
c) Specify the required rotation (in terms of the normal to the x-face) in order to achieve maximum shear stress in the xy face. Make a sketch of the xy face showing the components of stress and the angle rotated.



Problem 5.33

d) According to the material's properties, the failure criteria is given by a maximum stress of 4 MPa for shear and 6 MPa for normal stress. Predict if the material will fail due to the applied load. Why?