Time Value of Money Concepts: Uniform Series, Cap Value, and Payback Period

Module 02.3: TVM AE, etc.

Revised: January 27, 2003

Purpose:

- Expand TVM (time value of money) concepts into the development of other cashflow evaluation techniques besides NPV. Specifically:
  - Periodic series analysis
  - Capitalized Value (often called pro-forma or cap-value),
  - Payback period, and

Learning Objectives

- Students should be able to determine the NPV of a Bond.
- Students should be able to determine the Cap Value of a net revenue stream for a revenue generating asset.
- Students should be able to determine the Payback Period for a revenue generating asset.

A Review of Some Commonly Used Terms

- P, PV, and NPV – all mean Present Value or the value of the money Now.
- Now is time = zero.
- A "Cash Stream" a series of expenses and incomes over time. You "discount" a cashflow over time.
- F and FV stand for future value.
- A, AE, PMT all stand for the periodic amount in a uniform series or "annual equivalent" or equal installment payment, etc.
- Little "i" means interest rate; Big "I" stands for Interest amount. Watch for typos because PPT whimsically changes one to the other.

Three Kinds of Possible Problems

- Case 1: Time Finite, %>0 – Most real problems fall in this domain
- Case 2: Time Infinite, %>0 – Useful when you need a quick SWAG at NPV.
- Case 3: Time Finite, %=0 – Very useful where interest can be neglected for all practical purposes.

Case 1: Time Finite, %>0

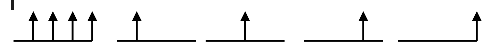
- Time is Finite and Interest Rate, i, is greater than zero. (The usual case.)
- You use this approach when a precise number is required.
- $P = F/(1+i)^n$ (We did this in Lecture 02.2)
- $P = A*((1+i)^n-1)/i*(1+i)^n$

Derivation of $F=P(1+i)^n$

Years	Start	Interest	End
First	P	iP	P(1+i)
Second	P(1+i)	iP(1+i)	$P(1+i)^2$
Third	$P(1+i)^2$	$iP(1+i)^2$	$P(1+i)^3$
n-th	$P(1+I)^{n-1}$	$iP(1+I)^{n-1}$	$P(1+i)^n$

QED, $F=P(1+i)^n$ OR $P=F(1+i)^{-n}$

Derivation of: F=A*(F/A,i,n)



(1) $F=A(1+i)^{n-1}+\ldots A(1+i)^2+ A(1+i)^1+A$

Multiply by (1+i)

(2) $F+Fi = A(1+i)^n+A(1+i)^{n-1}\ldots A(1+i)^2+A(1+i)^1$

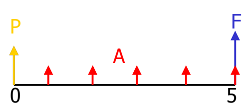
Subtracting (1) from (2), you get. $Fi=A(1+i)^n-A$

$F=A[((1+i)^n-1)/i]$, and

$P=A[((1+i)^n-1)/i(1+i)^n]$

Single Value Problem
Do in Class until everyone "gets it."

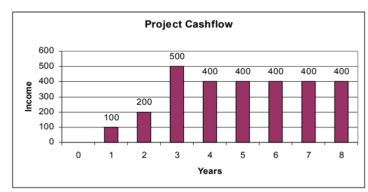
The relationships between equivalent amounts of money ($5,000 now) at different points in time are shown below.



1. P= $5,000, i=12%
2. F= $5,000(1.12)^5 = $8,811.71
3. A= $8,811.71*.12/(1.12^5-1) = $1,387.05
4. P= $1,387.05*(1.12^5-1)/(.12*1.12^5) = $5,000

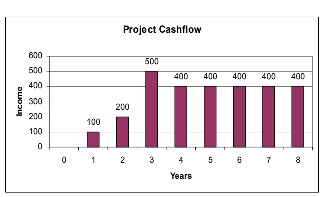
Example Cashflow

EOY	Amount
0	
1	100
2	200
3	500
4	400
5	400
6	400
7	400
8	400



Finding the Present Value using the factor method.

EOY	Amount	12%	PV
0		1.000	0
1	100	0.893	89
2	200	0.797	159
3	500	0.712	356
4	400	0.636	254
5	400	0.567	227
6	400	0.507	203
7	400	0.452	181
8	400	0.404	162
Sum	2,800		1,631



P= F*(1 +i)^-n

Now What's the Future Value using the factor method?

EOY	Amount	12%	FV
0			0
1	100	2.211	221
2	200	1.974	395
3	500	1.762	881
4	400	1.574	629
5	400	1.405	562
6	400	1.254	502
7	400	1.120	448
8	400	1.000	400
Sum	2,800		4,038

- F = P(1+%)^n
- F = $1,631*(1.12)^8 = $4,038

Now What IS the Annual Equivalent?

- $A = \$1,632*[.12*1.12^8 /(1.12^8 -1)]$
  $= \$328.52$
- $A = \$4,038*[.12/(1.12^8-1)]$
  $= \$328.21$

Bond Example

- This is usually called "discounted cash flow" and is easier than it looks,
- The only relationship you really need to know is: $P=F(1+i)^{-n}$
- But $P=A[((1+i)^n-1)/i(1+i)^n]$ helps
- For example, What is the PV of a 10-year, $10,000 bond that pays 10%, if current interest is 5%?

Bond Nomenclature

- The "face values" establish the cash stream to be evaluated at the current interest rate.
- A 10-year, $10,000 bond, paying 10% generates 10 equal payments of $1,000. The payments are at the end of the years.
- At the end of 10 years the $10,000 is also returned.
- The question is: What is the present value of that cash stream at 5% interest? At 15%?
- Note: the two different interest rates.

Bond Evaluation using Brute Force

	EOY	FV	Paymt	PV	5%
	0			13,861	
	1		1,000	14,554	
	2		1,000	14,232	
	3		1,000	13,893	
	4		1,000	13,538	
	5		1,000	13,165	
	6		1,000	12,773	
	7		1,000	12,362	
	8		1,000	11,930	
	9		1,000	11,476	
	10	10,000	1,000	11,000	

Bond Example Using P given A and P given F

- Break the problem into two parts: the series and the single payment at the end. Thus:
- $P_1=\$1,000[(1.05^{10}-1)/.05*(1.05)^{10} = \$7,722$
- $P_2=\$10,000*(1.05)^{-10} = \$6,139$
- At 5%, $P = P_1 + P_2 = \$13,861$
- At 10%, P =$6,145 +$3,855 = $10,000
- At 15%, P =$5,020 +$2,472= $7,491

Case 2: Time Infinite, %>0

- Time is assumed to be Infinite and Interest Rate, i, is grater than zero. (Cap Rate approach).
- This is good for a quick SWAG at finding the "value" of an asset from the cash stream that it generates.
- $P = A/i$, etc.
- Or $A = P*i$

Derivation of P=A/i

- There are two approaches
- Excel – strong-arm approach
- Math – Elegant
- $P=A[((1+i)^n-1)/i(1+i)^n]$
- Gets large without limit, everything cancels except for A/i.
- P=A/i or A=Pi or i=A/P

Cap Value Approach for Evaluating Rental Property

- CV = Annual Rent / Interest Rate
  CV = $6,000 / i=.085 = $70,000
  CV = $6,000 / i=10% = $60,000
- Use when i is your MARR (minimum attractive rate of return.)
- Notice that when MARR increases the price you should pay goes down.

Payback Period for Evaluating Rental Property

- CV = 120*Monthly Rent (Assumes a payback in 10 years.)
  CV = 120 * $500 = $60,000
  - Use when i is small, easy to borrow money
  - MARR is 10%.

Case 3: Time Finite, %=0

- Time is finite and short and Interest Rate, i, is equal to, or close to, zero.
- Used for a quick swag at complex problems.
- A = NPV/n
- or NPV = A*n

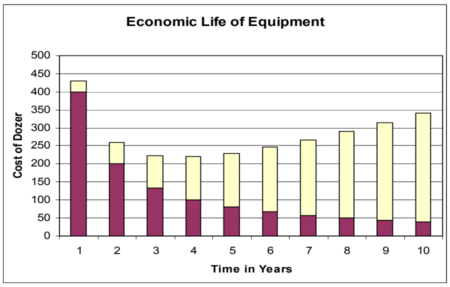
Case #3 Example: Economic Life

- Assume that a bulldozer costs $400k
- Assume that its O&M costs are $30k for the first year and increase $30k per year
- Then the cash stream looks like this:

Resulting Cash Stream

EOY	Ave Cost/yr	O&M/yr	Total
1	$400	$30	$430
2	$200	$60	$260
3	$133	$90	$223
4	$100	$120	**$220**
5	$80	$150	$230
6	$67	$180	$247

Plot of Cash Stream



Lecture Assessment

- Take 1 minute and Write down the one topic that is "muddiest" (least clear) for you.