TVM Basic Concepts: B/C Ratio, IRR, and ERR

Module: 02.4

Revised: February 7, 2003

Purpose:

- Expand TVM (time value of money) concepts into the development of other cashflow evaluation techniques besides NPV. Specifically:
  - B/C Ratio problems – Federally Funded
  - IRR & ROR – Internal Rate of Return - Banks
  - ERR – External Rates of Return

Learning Objectives:

- Given appropriate data and making appropriate assumptions students should be able to determine the:
  - B/C Ratio
  - IRR, and
  - ERR

For simple project cashflows.

B/C Ratio is important because … …

- Most federally funded project require evaluation by B/C Ratio.
- 1937 Law stated that the benefits to whomever they may accrue must exceed the cost to the government.
- Various interpretations of exactly what that means.

B/C Ratio

Conventional Equation

- B/C = Yearly Benefits / Yearly Costs to Gov.

Modified Equation

- B/C = Yearly Net Benefits / Yearly Net Costs

Note: every number is assumed to be annualized based on the same discount rate

B/C Ratio Example with i=0%

Every Benefit and Cost must be converted to an equivalent annual amount.

UB = 20 O&M= 5 IC= 100 Yrs= 10

B/C = UB / (IC/Yrs +O&M) = 1.3

B/C = (UB - O&M) / (IC/Yrs) = 1.5

B/C Ratio with i%>0

		AE
Initial Cost	$20,000,000	$3,116,402
Salvage Value	200,000	$13,164
Life = 10	10	
Interest = 9%	9%	
User Benefits	5,500,000	
O&M	$2,000,000	
Net Benefits	=User Benefits - O&M	
Net Cost	=AE of IC - AE of SV	
Modified B\C=	1.13	
Benefits =	User Benefits	
Costs =	AE of IC - AE of SV + O&M	
Conv B/C=	1.08	

The Costs are easy to quantify.

But what are user benefits?

Sometimes that's a judgment call.

RAT #02.4.1

- Pull out a sheet of paper.
- Individually work in 5-minutes:Given a project life of 50 years, an Initial Cost of $25-million, annual benefits are $1.25-million and O&M costs are $500k. Neglecting interest costs and salvage value, what is the B/C ratio using the modified formula?

Typical Replacement Problem, B/C with i<>0%, n is finite

Three alternatives (A, B, and C) have been suggested to **replace** the current situation. Each of the alternatives has an expected life of 20 years with negligible salvage value. Use a 9% discount rate. Which would you pick using B/C ratio?

i=9%, n=20yrs	Current	Project A	Project B	Project C
Initial Cost ($)	$0	$100,000	$175,000	$225,000
Annual O&M Cost	$15,000	$8,000	$10,000	$25,000
Annual Benefits	$20,000	$30,000	$35,000	$50,000

The basic approach is to evaluate the differences between the current and each suggested alternative.

Step #1: Convert everything to annual using the A=P(A/P,9,20) relationship.

Step #2: Compute deltas

Step #3: Compute B/C and pick "best."

i=9%, n=20yrs	Current	Project A	Project B	Project C
Initial Cost ($)	$0	$100,000	$175,000	$225,000
Annual O&M Cost	$15,000	$8,000	$10,000	$25,000
Annual Benefits	$20,000	$30,000	$35,000	$50,000
Conventional B/C Ratio				
Modified B/C Ratio				

Conv B/C = Delta Benefits / (AC of IC – Delta O&M)

Mod B/C = (Delta Benefits + Delta O&M) / AE of IC

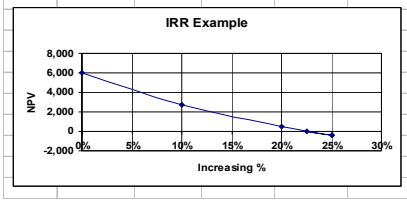
i=9%, n=20yrs	Current	Project A	Project B	Project C
Initial Cost ($)	$0	$100,000	$175,000	$225,000
Annual O&M Cost	$15,000	$8,000	$10,000	$25,000
Annual Benefits	$20,000	$30,000	$35,000	$50,000
A/E of IC		$10,955	$19,171	$24,648
Delta O&M	As Benefit	$7,000	$5,000	($10,000)
Delta Benefits		$10,000	$15,000	$30,000
Conventional B/C Ratio	Reduced O&M Reduces Cost	2.53	1.06	0.87
Modified B/C Ratio	Reduced O&M Adds Benefits	1.55	1.04	0.81

Rate of Return

- In essence any cash stream may be represented by a polynomial:
- PV = A_0 +A_1*(1+i) +A_2*(1+i)^2, … A_n()^n where A is any amount of $$'s
- The Internal Rate of Return (IRR) is that interest rate that causes the value of the equation to go to zero.
- Excel has a function that will find it, or "Trial and Error" as suggested by Halpin.

Trial and Error IRR Calculation

EOY	Amount				
0	-8,000	-8,000	-8,000	-8,000	-8,000
1	2,000	1,818	1,667	1,600	1,633
2	3,000	2,479	2,083	1,920	1,999
3	4,000	3,005	2,315	2,048	2,176
4	5,000	3,415	2,411	2,048	2,220
%	0%	10%	20%	25%	22.50%
NPW	6,000	2,718	476	-384	28



Some Hints.

- In math terms the "curve" decreases monotonically to the right to cross the X-axis. Therefore,
- NPV at 0% must be a positive number. So check that first.
- If NPV at 0% is a large number, IRR must be large so pick you first % at 20% or more to bound the answer.

RAT #02.4.2

- Pull out a sheet of paper and get your calculators ready.
- Take 5-minutes to compute the IRR of the cash flow table to the right.

EOY	Amt
0	$ (6,000)
1	$ 2,000
2	$ 2,000
3	$ 4,000

RAT #3.3.2, Continued

- Check with your Pair and see if you got the same answer? If not why not? Take 2-minutes.
- Now do the same with within Teams.
- Team ? What answer did you get? Etc.

Some More IRR Issues

- As you sum amounts from left to right, the cashflow equation changes sign at least once. Ill behaved cash flow may change sign more that once, indicating multiple roots.
- Ill behaved cashflows may approach the X-axis asymptotically implying an improbably large IRR.

External Rate of Return

- Works for all types of cash flows – messy or otherwise.
- The Steps:
  1. Discount all expenses to the Present at the "prime" (what you have to pay) interest rate.
  2. Project all income to the Future at the "T-Bill" (what you can get) rate.
  3. Consolidate numbers and Solve resulting $P=F/(1+i)^n$ for i

Example ERR

Year	Cash Flow	PV@8%	FV@5%
0	10,000	10,000	
1	15,000	13,889	
2	0		0
3	1,000		1,407
4	2,000		2,680
5	4,000		5,105
6	6,000		7,293
7	7,000		8,103
8	6,000		6,615
9	5,000		5,250
10	4,000		4,000
5.41%	10,000	23,889	40,454

- Assume:
  - 8% borrowing
  - 5% earning
  - Ten Year Project
  - Cashflow as shown

RAT #02.4.3

- Pull out a sheet of paper, get your calculators ready … …
- First As Individuals take 2 minutes to:
- List the steps necessary to perform a ERR analysis.
- Then as pairs take 2 minutes to:
- Combine and correct lists.

RAT 02.4.3, Continued

- Now Individually work the problem.

Year	Cash Flow		
0	10,000		
1	15,000		
2	0		
3	1,000		
4	2,000		
5	4,000		
6	6,000		
7	7,000		
8	6,000		
9	5,000		
10	4,000		
	10,000		

Assume:

- 10% borrowing
- 5% earning
- Ten Year Project
- Cashflow as shown

When to Use Each Technique?

- It depends on what you are evaluating and who you are trying to convince.
- NPV adds to the "balance sheet" and the total worth of the organization or measures the value of an investment for "pricing" issues.
- B/C is required for most all Gov. projects.
- IRR% is how "investors" look at things. ERR% is more realistic but … …
- AE is how operators look at things and operating cashflow issues.

Class Assessment

- In one sentence, which topic do you think needed to be covered in more detail?