

Experimental Tests of Force and Acceleration

By

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Executive Summary

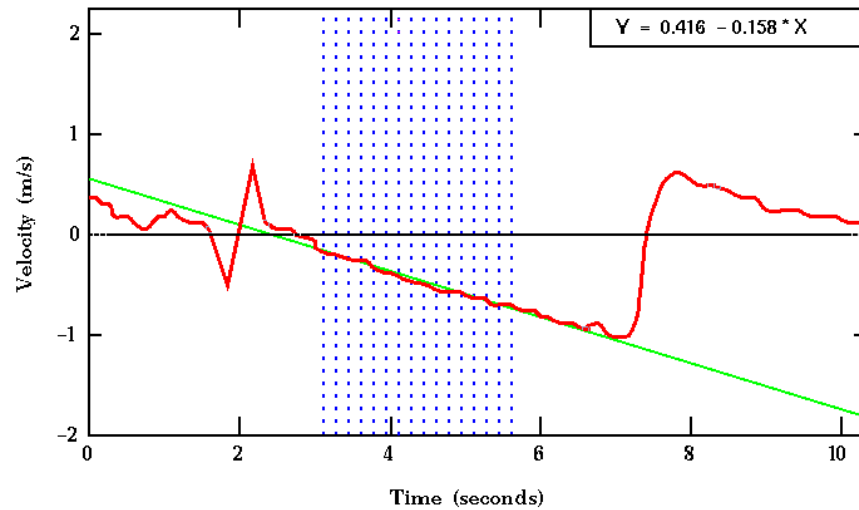
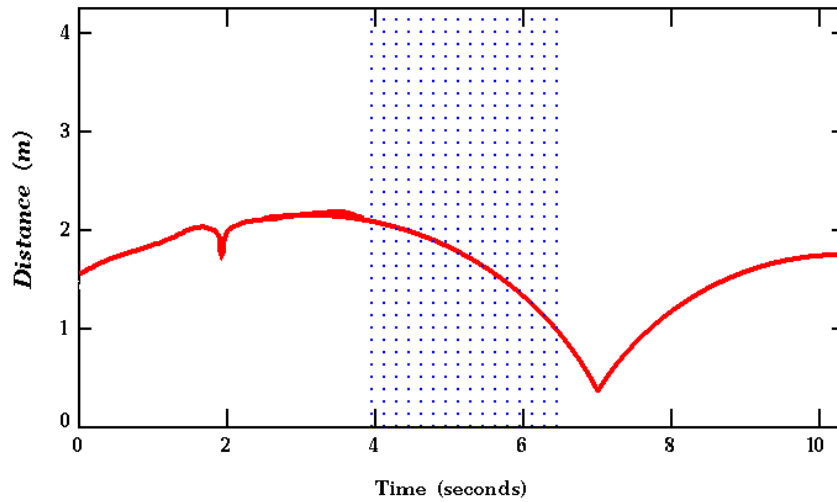


Data and Calculations

We measured the acceleration due to gravity of a glider on an inclined air track. The mass of the glider (including the reflecting flag) was 0.218 ± 0.001 kg. The separation of the supports under the glider is 1.0 ± 0.001 meters. The inclination angle was fixed by placing one or more spacers (each of height 0.008 meters) under one of the air track supports. We used the ultrasonic ranger to determine position (and velocity) of the glider as a function of time; the acceleration is the slope of the velocity vs time graphs. Four measurements (referred to as trials in the table below) were made for each angle of inclination. The uncertainty in the measurement of track elevation and track length was 0.001 meters.

Trial	Height (meters)	Acceleration (m/s^2)
1	0.008	0.077
2		0.080
3		0.076
4		0.080
5	0.016	0.158
6		0.152
7		0.160
8		0.161

Representative graphs (trial 5) of distance and velocity as a function of time are shown below.



From the data shown, we find that the average accelerations for the 0.008 meter and 0.016 meter heights are $0.0785 \pm 0.002 \text{ m/s}^2$ and $0.158 \pm 0.003 \text{ m/s}^2$, respectively. The uncertainty is estimated from the spread of the data points. We compute g from the acceleration data using the relation $a = g \sin(\theta)$, the acceleration of an object on a frictionless incline at angle θ to the horizontal. The angle of inclination, θ , for height, h , and 1 meter separation of the air track supports is $\theta \sim \tan(\theta) = h/1 = h$. (The small angle approximation is valid here.) Our uncertainty on the measurement of θ is derived by summing the percentage errors in our length measurements. The percentage error for h and for the nominal one meter length used for angle determinations were 10% and 0.1%, respectively. Hence, the percentage error in θ is about 10%. Our values for g for each trial are:

Trial	Inclination angle (radians)	value of g (m/s ²)
1	0.016	9.625
2		10.00
3		9.500
4		10.00
5	0.016	9.875
6		9.500
7		10.00
8		10.06

Instead of propagating errors mathematically, we averaged our results and estimated the error from the spread in values around the average. For the data shown in the table, we find $g = 9.82 \pm 0.2 \text{ m/s}^2$

We next added mass to the glider in increments of 100 grams and again made several measurements of acceleration for each of our two heights. The analysis of these trials follows exactly as described above for the unweighted glider. In each case, the uncertainty on the glider mass is 0.001 kg and the uncertainty on the determination of g is 0.2 m/s², or about 2% fractional error. Our results for all measurements are summarized below:

Trial	Glider mass (kg)	value of g (m/s ²)
1	218	9.82
2	318	9.80
3	418	9.79

We discuss the significance of these results in the next section.

Analysis

The best estimate we have found for the value of **g** in Philadelphia is $9.802 \pm 0.001 \text{ m/s}^2$ (reference *Physics* by Eugene Hecht, pg. 73). According to Newtonian theory, this value should be the same for all objects regardless of mass. Our result for three different glider masses shows that **g** is independent of mass and are consistent within their 2% uncertainty. The average value of **g** from all three masses is 9.80 ± 0.01 . The uncertainty on **g** is obtained by forming the quadratic sum of the uncertainty on each measurement of **g** (0.02 m/s^2). Our results are consistent with the value given in Hecht. There are several sources of uncertainty in the determination of the gravitational acceleration: the measurement of lengths with a meter stick; 2) the intrinsic measurement accuracy of acceleration with the MacMotion program; and 3) the influence of air resistance on the glider. The measurement of the length between the air track supports is appropriate for the meter stick since the length to be measured is comparable to stick length and the fractional error is small. However, using the same stick to measure the small height of the blocks used for inclination leads to a much larger fractional error. These lengths would be better measured using calipers or a micrometer. The uncertainty in the height accounts

for essentially all of the uncertainty on the determination of the inclination angle. Since the angle stays fixed for several trial measurements of the acceleration, this error does not play a role in determining the spread of acceleration values.

The uncertainty in the determination of acceleration accounts for essentially all of the spread in our values of g . This spread is presumably partially due to actual small differences between the trials and partially to limitations in the instruments and software. Fitting various parts of a section of good data and repeating this for several data plots indicate that we have 2% uncertainty in our acceleration measurements, which leads to the 2% spread in our determination of g for a fixed incline angle. Some method for directly determining velocity, e.g. with direct timing measurements over short intervals might reduce this uncertainty.

Finally, air resistance needs to be reduced. Air resistance gives us lower values of acceleration than we would otherwise measure, hence it should tend to push our measured values of g lower than the true value. It is difficult to measure the extent of this effect. We see little evidence for it in our measurements, presumably because air resistance increases as the speed of the glider increases and all our velocities for small inclination angles are small. This effect might be minimized by switching to an ultrasonic reflection flag which is streamlined.

Conclusions

The determination of the independence of gravitational acceleration on mass or content of a falling body represents a major achievement in scientific thought. It allows the use of gravitationally accelerated objects to be used in testing concepts of inertia and the relationship between force, mass, and acceleration. In addition to verifying Galileo's original observation, our experimental technique can be used to measure g to about 1% in any local environment. We have suggested methods by which even more accurate values of gravitational acceleration may be determined.