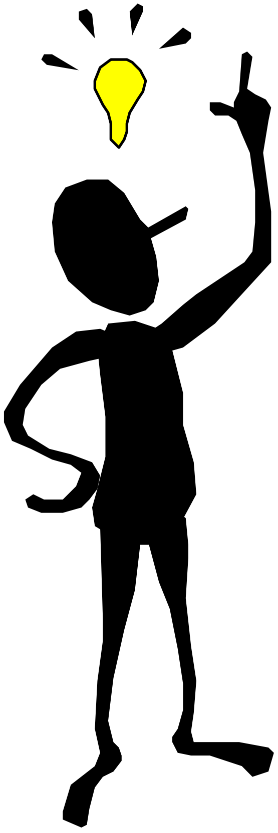
Engineering Modeling Skills:
Period 3

Review

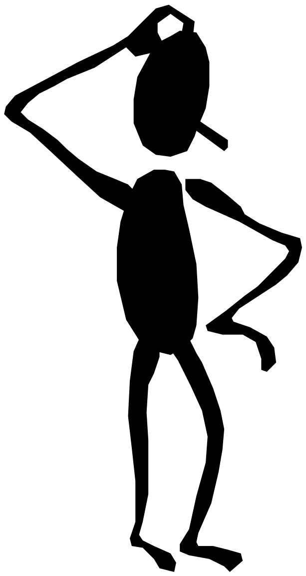


Engineering analysis, design, and optimization require that processes, products, or systems be described quantitatively. Therefore, engineers must express problems or concepts in mathematical language — as some type of

model.

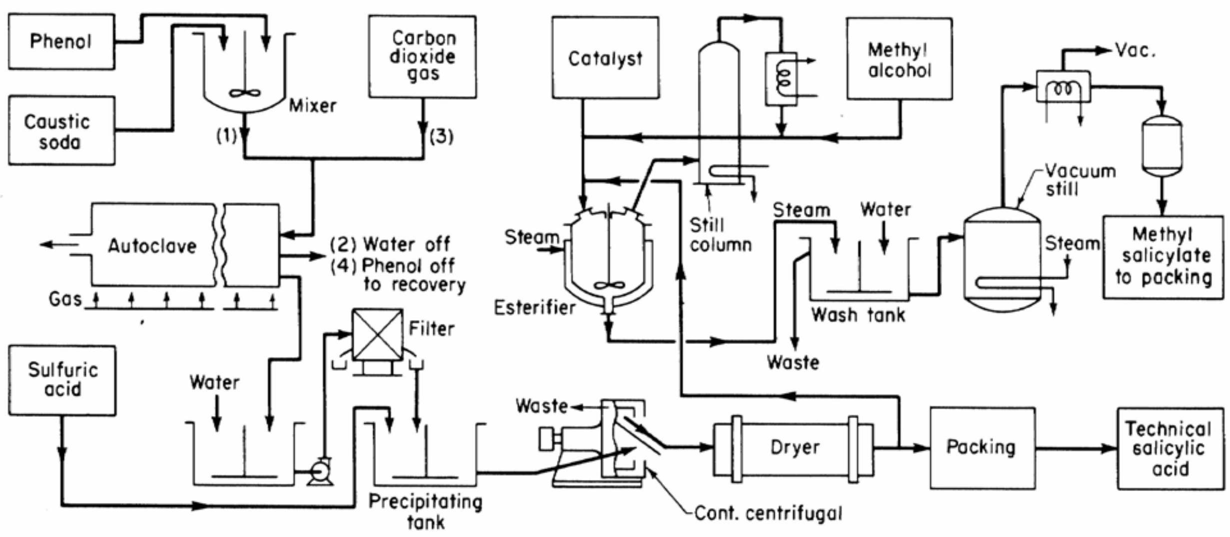
Review

How do we create models?



- What result is needed?
- System abstraction
- Identify fundamental principles
- Mathematical formulation
- Solve formulation
- Test the results
- Generalize
- Simplify

As processes and systems get more complicated, so do the models that are used to describe them. For example, shown below is a simplified schematic of part of the process used to make aspirin. A standard model for this process involves hundreds of simultaneous, nonlinear differential equations.



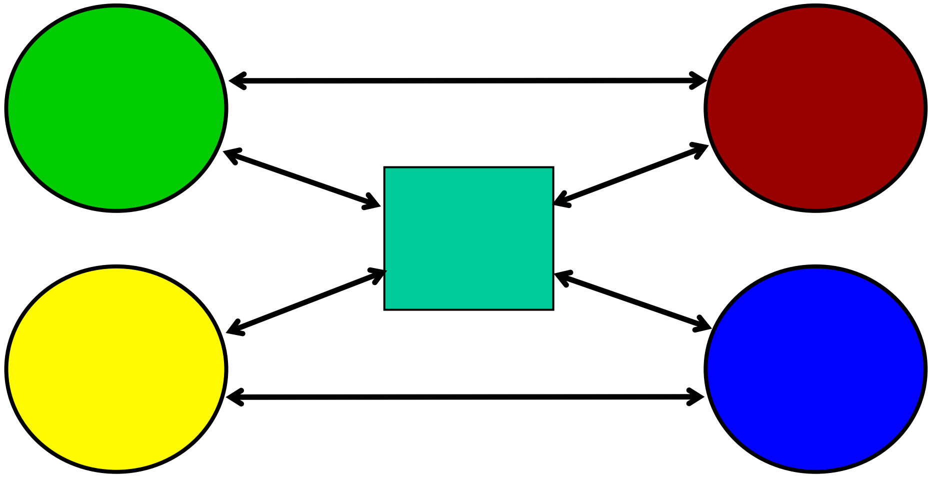
Isn't engineering fun!

However, it is for these processes and systems that modeling becomes most valuable. We don't want to spend thousands, millions, or billions of dollars to build a new integrated circuit, chemical plant, or spacecraft only to discover after the fact that 'things don't work.'

So, as models become more crucial, they also become more difficulty to handle — rapidly approaching intractability.

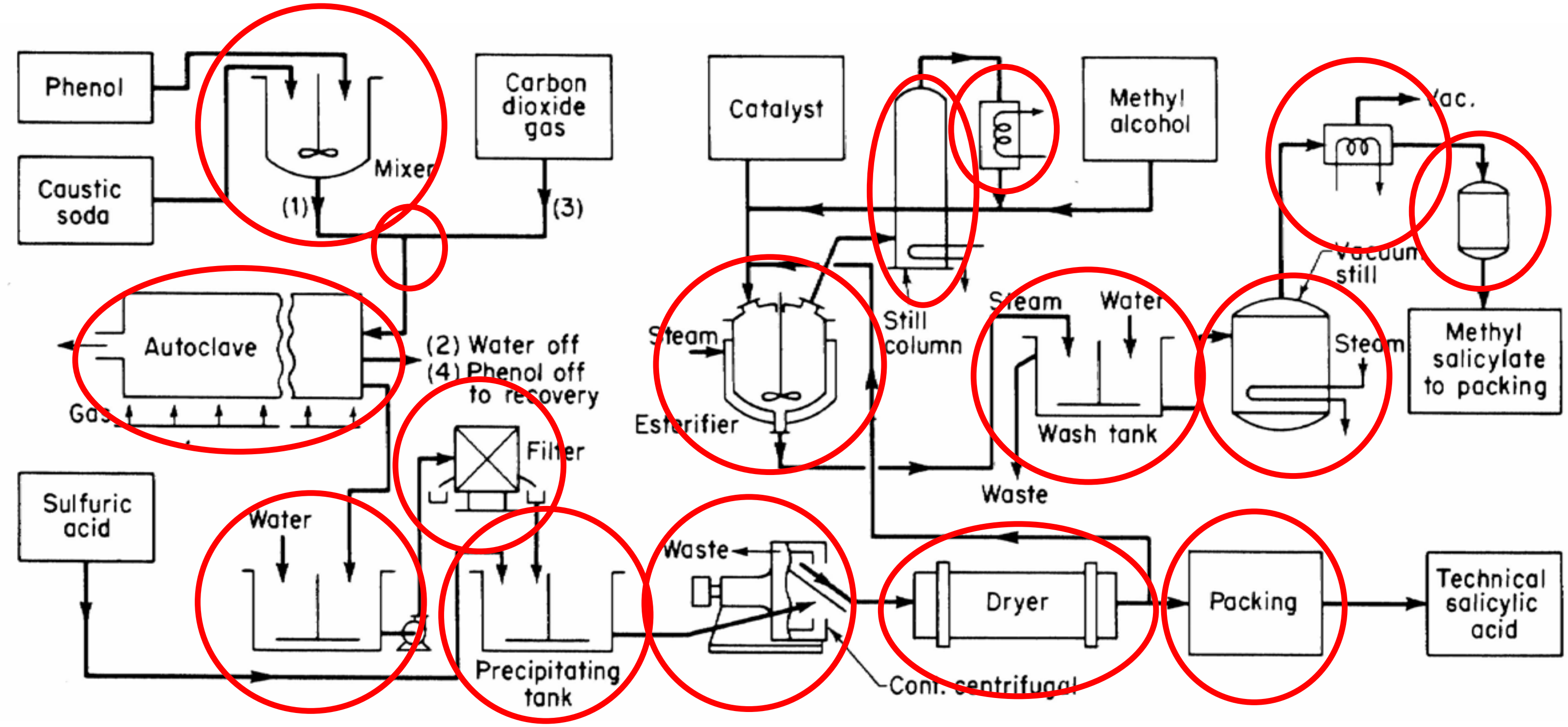
What are we to do?

The modeler's solution to this dilemma is to break the process or system into smaller pieces (modules) — each of which can be modeled individually — with information flow between the modules. In analogy with computer programming, this technique can be thought of as *object oriented modeling*.

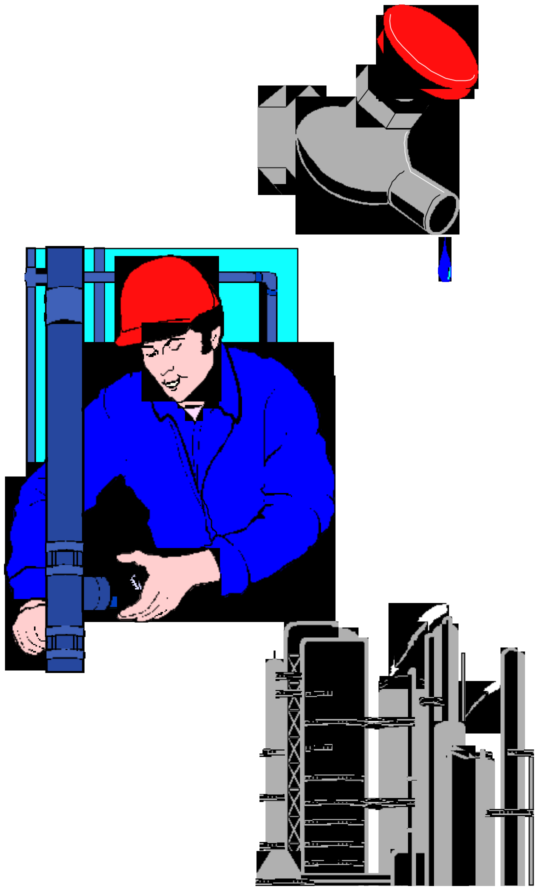


Deciding how to break the process or system into modules is part of the art of modeling, but some possible strategies include location, geometry, functionality, or interaction.

For example, the aspirin process shown earlier can be divided based on interaction (exchange of material) as:

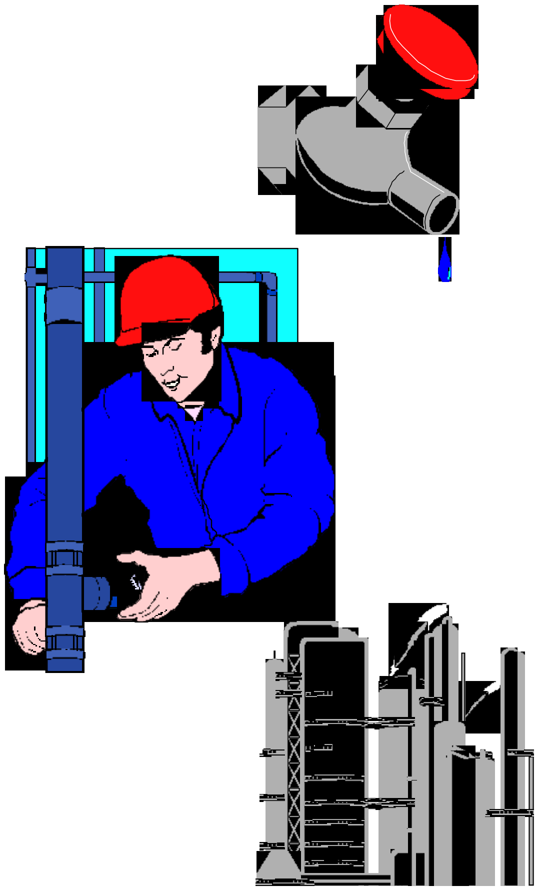


Example: Pipe Network

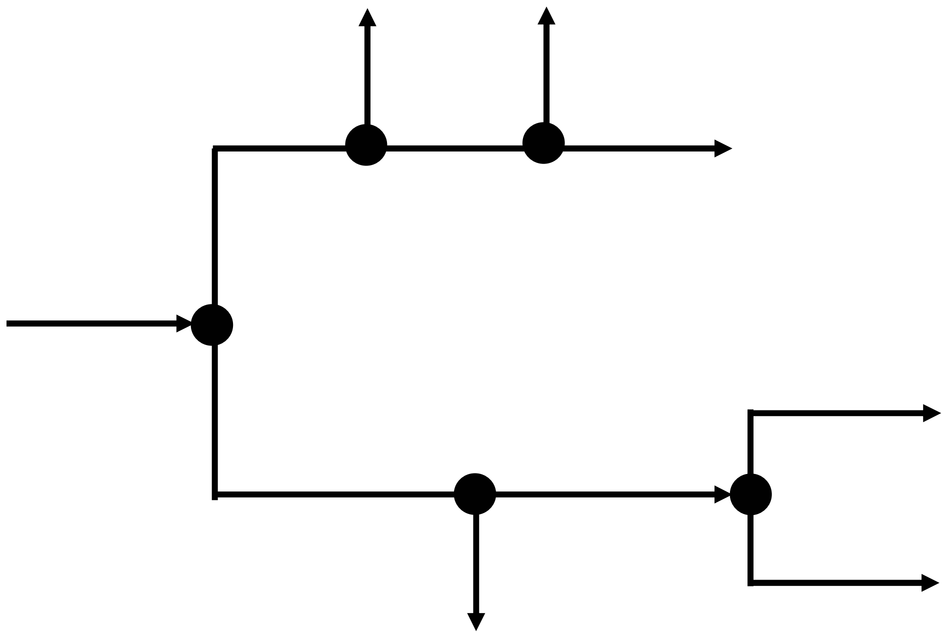


Fluids handling systems (*i.e.,* pipe networks) can get very complicated. Imagine a municipal water distribution system. Many pipes, pumps, valves, and fittings in a variety of sizes are involved. Nonetheless, the individual components in such systems are quite simple and easy to model — only simple conservation relations (mass, momentum, and energy) are needed.

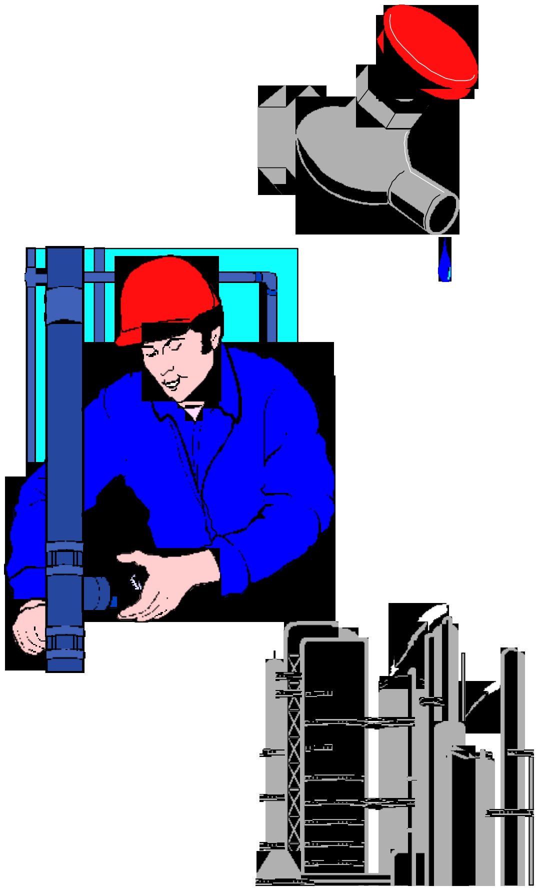
Example: Pipe Network



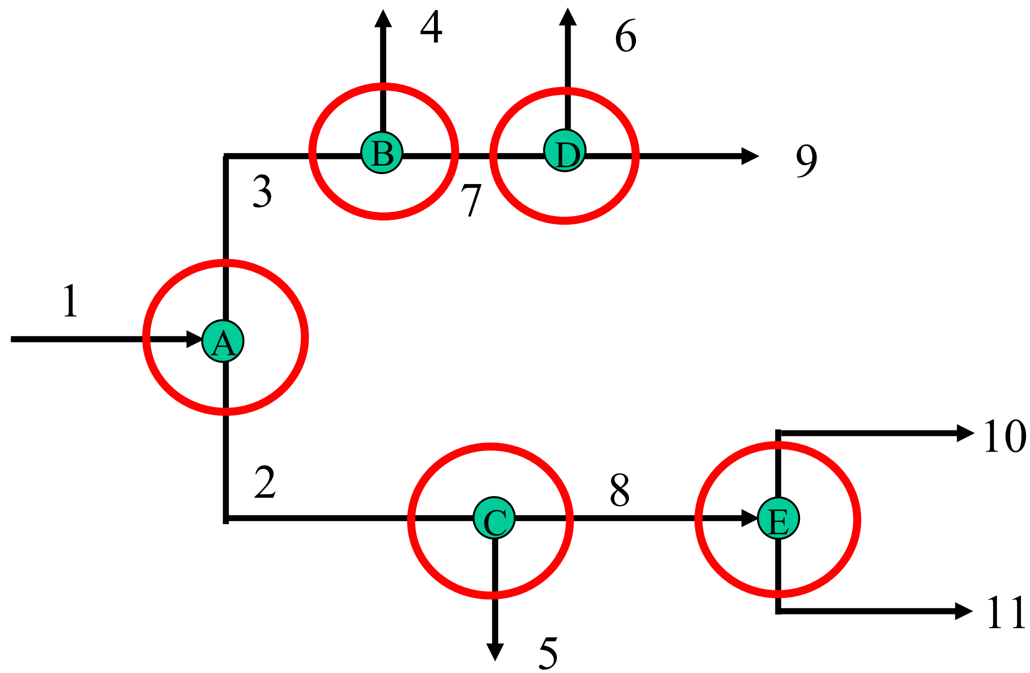
Consider the simple system sketched below (solid lines represent pipes, and circles represent fittings):



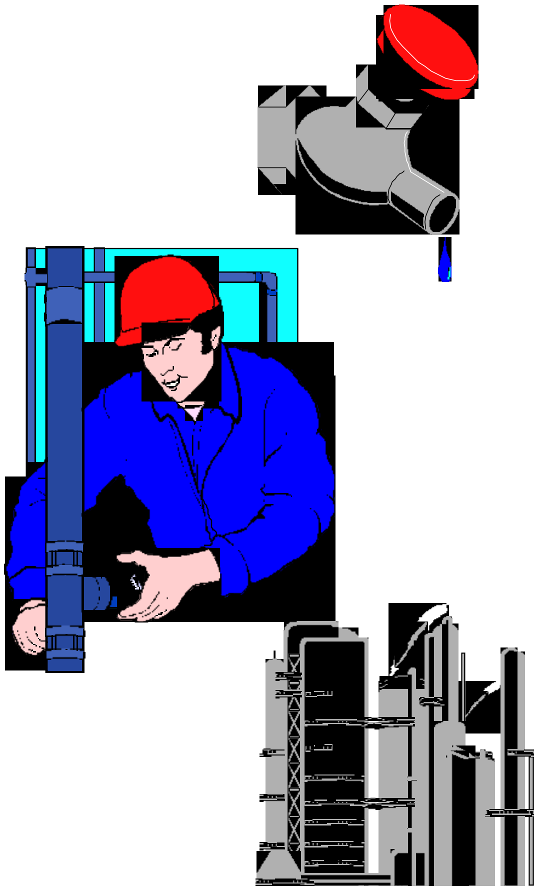
Example: Pipe Network



We can identify the streams (numbers) and divide the network into components (letters) as:



Example: Pipe Network



The individual components are simply modeled using conservation of mass. That is, the total mass flow rate into a component must be equal to the total mass flow rate out. Additional relations arise describing the extent to which components split the flow — we assume that each component divides the entering stream into equal outlet streams. Mathematically, these relations are:

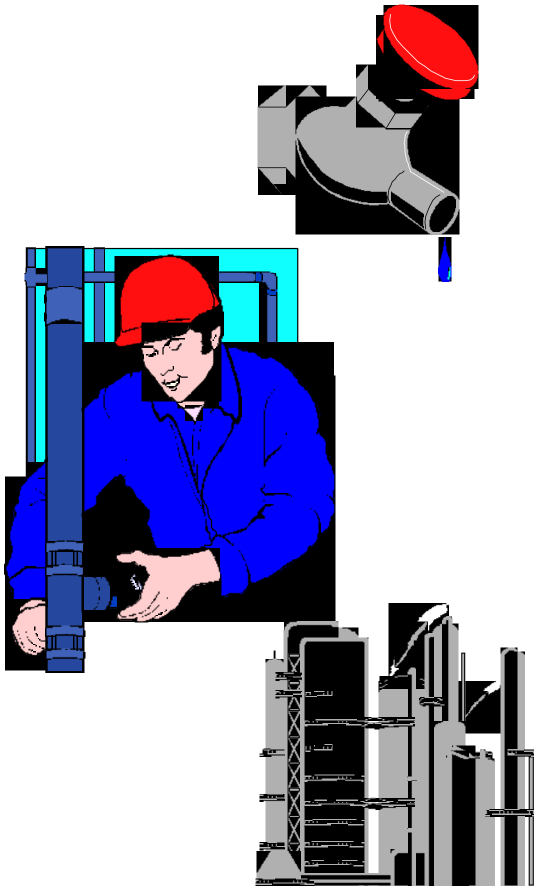
$$\sum_{inlets} \dot{m}_{in} = \sum_{outlets} \dot{m}_{out}$$

$$\dot{m}_{out} = \frac{1}{2}\dot{m}_{in}$$

Where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the mass flow rates of the entering and leaving streams from the component.

Disclaimer: A more realistic model would describe the pressures in the system and would require the use of momentum balances.

Example: Pipe Network



We write these relations for each of the system components to obtain an overall model involving N=10 equations and M=11 unknowns:

A	$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ $\dot{m}_2 = (1/2)\dot{m}_1$
B	$\dot{m}_3 = \dot{m}_4 + \dot{m}_7$ $\dot{m}_4 = (1/2)\dot{m}_3$
C	$\dot{m}_2 = \dot{m}_5 + \dot{m}_8$ $\dot{m}_5 = (1/2)\dot{m}_2$
D	$\dot{m}_7 = \dot{m}_6 + \dot{m}_9$ $\dot{m}_6 = (1/2)\dot{m}_7$
E	$\dot{m}_8 = \dot{m}_{10} + \dot{m}_{11}$ $\dot{m}_{10} = (1/2)\dot{m}_8$

If we specify M-N=1 of the flow rates, this system of linear algebraic equations can be solved by any of a variety of methods (see today's assignment).

Degrees of Freedom Analysis

It is important to know if a mathematical problem can be solved — particularly for more complicated models. At the most basic level, this involves calculating the number of degrees of freedom in the model: the number of unknowns minus the number of equations. If this number is equal to zero, the model is completely specified. If the number is negative, the model is over-specified, and we need to return to the Abstraction or Formulation steps to see where we went wrong. If the number is positive, then the model is under-specified. This last case is not as bad as it might seem. It simply means that we need to specify more information.

For models that involve differential equations, the number of boundary and initial conditions must also be consistent with the number and order of derivatives appearing in the model.

Discipline Specific Modeling Tools

Engineers must:

- Formulate engineering problems or concepts as mathematical models.
- Identify assumptions invoked in or implied by models.
- Specify constraints and conditions on models.
- Be aware of the limitations of models.

Steps for Building Models

- What result is needed?
- System abstraction
- Identify fundamental principles
- Mathematical formulation
- Solve formulation
- Test the results
- Simplify
- Generalize

Homework:

Work in groups of 4 or fewer.

Implement on Excel a model for the piping system described in class today.

Allow the entering flow rate, $\dot{m}_1$, to be varied in the spreadsheet.